Conditions for Total Internal Reflection of Radiofrequency Wave in Optic Fiber for Efficient Communication

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Abstract
The transmission of signal from the sender to the receiver for effective communication is guided by a phenomenon of physics known as total internal reflection (TIR) which must strictly obey certain rules. This phenomenon occurs when incident radiofrequency wave hits the interface between the fiber core and cladding at an angle greater than the critical angle, causing the signal to be reflected back into the core. This research provides suitable wave equation model with appropriate boundary condition for total internal reflection. The exploration of ultra-critical angle, refractive indices, and fiber design parameters that facilitate efficient signal propagation are studied for effective communication. Understanding these conditions is crucial for designing and optimizing optical fiber communication systems. This study utilized theoretical mathematical model of wave equation and adoption of Snell's law in cylindrical coordinate system, refractive indices in the core and cladding, which determines the behavior of light when propagating from transmitter to receiver through the fiber are studied. The exact angles of incident signal are determined for TIR in line with corresponding refractive indices at appropriate direction of propagation of the signal within the system. It was found that incident angles of transmitted radiofrequency wave must not only be greater that than the critical angle but must be integral multiples of Pi $\theta_i = \pi, 2\pi, 3\pi, \ldots,$ otherwise there would be partial exit through the cladding by a phenomenon known as refraction would occur. It is recommended that this study be carefully considered when designing optical fiber system.

Keywords: Total internal reflection (TIR), refractive index, Snell's law, radiofrequency, optical fiber, ultra-critical angle.

Introduction

Whenever any of the electromagnetic waves in the electromagnetic spectrum interacts with matter any of the characteristics (Reflectance, Absorbance and Transmittance) is/are exhibited singly, or any combination pair, or the combination of the three.

When light travels through space it interacts with matter during its course of propagation where this interaction could be characterized based on the nature of matter/materials and the incident ray (Seeds & Williams, 2006). The interaction could be absorption, reflection or transmission, or any combination of the three.

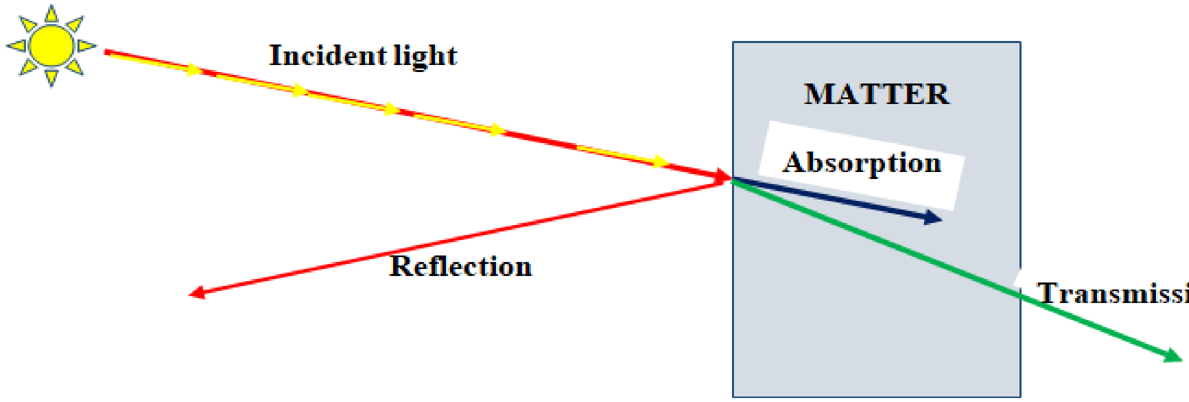

Fig 1: Resulting Properties from Interaction of Light with Matter (Jayeoba, et al., 2023)

The overall summation of any possible interaction behaviour is equal to unity.

Transmittance + Reflectance + Absorbance = 1 (1)

A very important application of this physics is in communication where an encoded radiofrequency wave interacts with a pair of transparent medium with different density in order to transmit and receive information; that is, involvement of transmission of RF signals over optical fiber links. For effective communication where signals are not significantly affected by factors such as attenuation, dispersion, and nonlinear effects, there are many optical, physical, structural and electrical design parameters on which the signal must be transmitted from end to the other of the link (Kaminow & Willner, 2013). Few among these parameters are: signal encoding and decoding processes (modulation and demodulation), transmitter and receiver materials characteristics, optic fiber cable characteristics (such as high bandwidth capacity, allowing for the transmission of high-speed RF signal), electromagnetic interference immunity (EMI). As important as the aforementioned parameters, there is a very essential underlying physics with limited understanding, and this is total internal reflection (TIR) where it is fundamentally ascertained that the incident ray from one optically dense medium to less dense medium must take place at an angle greater than the critical angle (Hetch, 2018). It is of utmost importance for the incident radiofrequency wave to be totally internally reflected within the medium of communication (optic fiber) to avoid signal attenuation during transmission, a major problem that may results in loss of lives and valuable properties.

Where an encoded RF wave passes through pair of transparent medium, a combination of refraction and reflection phenomena are liable when the direction of propagation is from dense medium to less dense medium (Keiser, 2010). These two properties are traded off individually as a function of incident angle in the first medium. In communication industries, the reflected rf wave is more valuable especially when it is purely totally internally reflected, a situation where there is no refraction (Wake, Nkansah, & Gomes, 2010).

Methodology

The relationship between the reflected and refracted rays can be determined experimentally in line with Snell's law for systems where the angle of incidence for light traveling from optically dense medium to less dense medium such as from glass to air is gradually increased. Figure (2) shows the experimental setup for transition of light from dense to less medium (Nelkon & Parker, 1970).

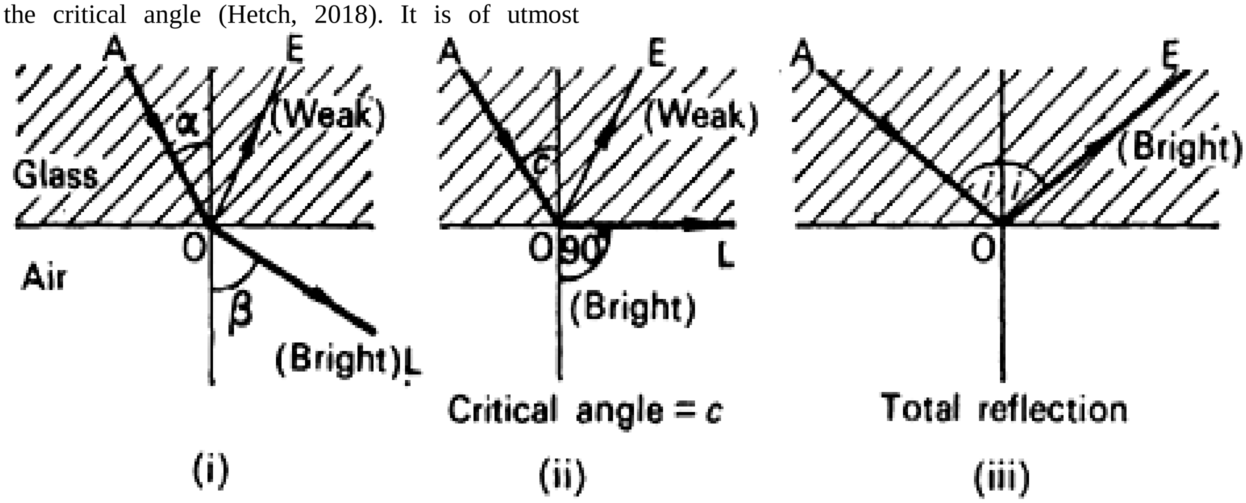

Figure 2: Behavioral Characteristics of light transition from dense to less dense medium

For the system describes in Figure (2), where light is incident from dense medium (glass) into a less dense medium (air) two major properties of incident light are altered as a function of density. The mathematical characteristics of systems (i) and (ii) are summarized using Snell's law.

<u>Characteristics of System (i):</u>

The reflection is weak, refraction is bright and Snell's is validly applicable to the system as

$$\frac{\sin\alpha}{\sin\beta} = \frac{1}{n} \quad (2)$$

where α is the angle of incidence, β is the angle of refracted ray with respect to the normal of the medium and n is the refractive index of the medium.

Characteristics of System (ii):

The reflection is weak, refraction is brighter and Snell's is validly applicable to the system as

$$\frac{\sin c}{\sin\beta} = \frac{1}{n} \quad \Rightarrow \quad \frac{\sin c}{\sin 90^0} = \frac{1}{n}, \quad \Rightarrow \quad n = \frac{1}{\sin c} \quad (3)$$

where c is the angle of incidence (critical angle) greater than α that gives 90° angle of refraction, β and n is the refractive index of the medium. This means that the refractive index of a pair of optical medium cannot be zero.

With Figure (2) experimentally, it has been concluded that system (iii) gives rise to TIR at an angle of incident greater than the critical angle, but this is not a sufficient condition for TIR for effective communication. Therefore, considering the wave equation in cylindrical coordinate system as shown in equation representing the RF signal (4):

$$\frac{\partial^2 y(r,\theta,z,t)}{\partial t^2} = c^2\left[\frac{\partial^2 y(r,\theta,z,t)}{\partial r^2} + \frac{1}{r}\frac{\partial y(r,\theta,z,t)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 y(r,\theta,z,t)}{\partial \theta^2} + \frac{\partial^2 y(r,\theta,z,t)}{\partial z^2}\right] \quad (4)$$

where c is the speed of light in a vacuum

The passage of rf signal within the optic fiber can be model for effective communication where the total incident signal from the transmitter passes through the cable without refraction that causes signal attenuation. Considering the situation where incident signal is sent at angle greater than the critical angle for TIR to occur. Figure (4) shows the optic fiber model for communication.

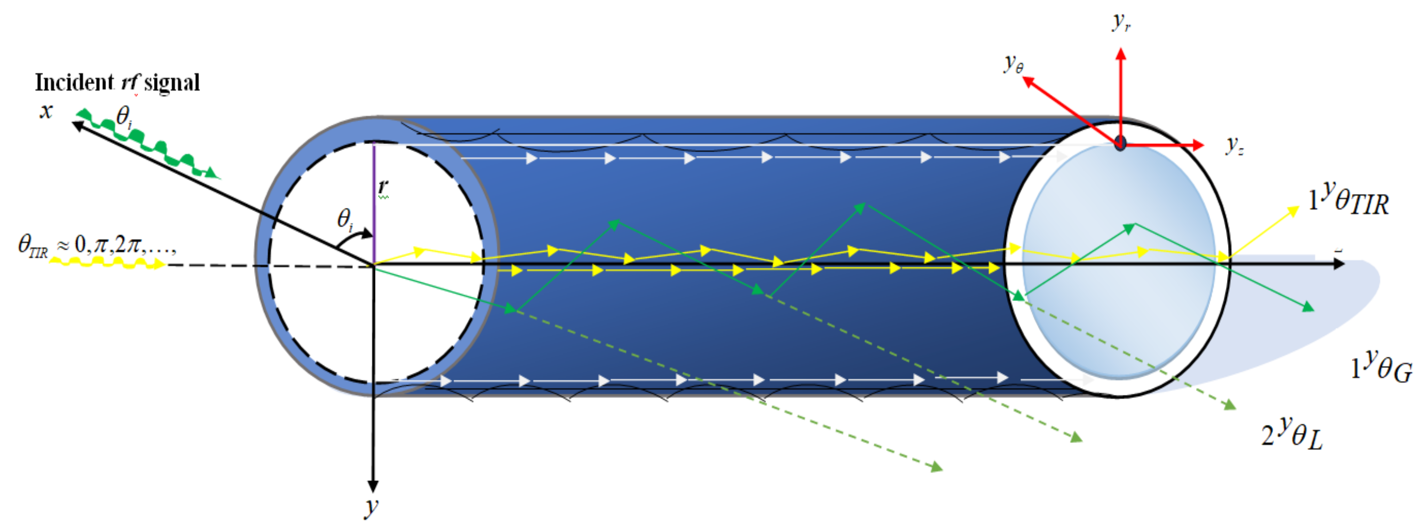

Figure 3: RF signal model in cylindrical coordinate system for communication

For a plane polarized rf incident signal travelling along *z*-axis of cylindrical coordinate, the incident ray angularly vary with the direction of propagation (*z*-axis) beyond the critical angle to fulfil the condition for total internal reflection.

Since there is no motion along the ***r***-direction for a plane polarized incident ***rf*** wave equation (4) is reduced to equation (5):

$$\frac{\partial^2 y(\theta,z,t)}{\partial t^2} = n^2 v^2 \left[\frac{1}{r^2} \frac{\partial^2 y(\theta,z,t)}{\partial \theta^2} + \frac{\partial^2 y(\theta,z,t)}{\partial z^2} \right] \qquad (5)$$

where $n = \frac{c}{v}$, is the refractive index, v is the speed of light in the optic fiber.

Equation (5) represents a typical the incident rf signal that is used to characterize system (iii) with sufficient boundary condition for TIR.

Characteristics of System (iii):

Considering a situation where the angle of incident ray is varied and reaching points beyond the critical angle, it is found that a pair of transparent media no longer exhibits the phenomenon of refraction but only a phenomenon known as Total Internal Reflection (TIR).

The reflection is bright, no refraction and Snell's is not valid for this system since there is no refraction because all the incident rays are total internally reflected (TIR).

The following summary can be derived from the system:

- Angle of incidence is greater than the critical angle,
- There is no refraction, (i.e. angle of refraction, $r = 0$),
- The reflection is bright (Total internal Reflection),
- The Snell's equivalent is given as $\frac{\sin i}{\sin r} = \frac{1}{n}$, since $r = 0$, then Snell's law is invalid.

From Snell's law, equation (2) $n \cdot \sin i = \sin r$

$$r = 0 \quad \Rightarrow \quad n \cdot \sin \theta_i = 0 \quad (6)$$

Since zero refractive index is not physically possible, as it would imply that the material has no optical properties, $n \neq 0$.

$$\therefore \sin \theta_i = 0 \quad (7)$$

Equation (7) gives the condition for TIR in line with the model in Figure (3), where there would be no signal loss due to refraction.

$$\sin \theta_i = 0 \text{ when } \theta_i = a\pi$$

where $a = 0,1,2,3,..........,\alpha$

(8)

Results and Discussion

In optic fiber, the input signal (rf wave) is one of the electromagnetic wave present in the electromagnetic spectrum with application in communication and telecommunication industries. So it can be modeled with the wave equation in cylindrical coordinated system in equation (4).

For optic fiber signal, the rf signal is plane polarized, where there is no signal propagation along the radial direction (*r*-direction) of the system. Coupled with light travelling through the optic fiber of specific refractive index, this shows the medium is not a vacuum. Therefore, this study modifies equation (4) to (5).

With equation (5) representing the incident rf signal and considering system (iii) where it has been established that with angle of incident greater than the critical angle is not a sufficient condition for TIR, and Snell's is known to be invalid for this system since there is no refraction.

Snell's law is modified as an extension to system (iii) and discovered that the product of sine of angle of incidence and the refractive index must be equal to zero for TIR equation (6). Since refractive index of an optical material cannot be zero, then sine of angle of incidence is zero. with this it was found that the angle incidence must not only be greater than critical angle but must be at integral multiple of Pi (π) as in equation (8).

Considering and combining the conditions, this study has the prime conditions summarized as follows:

$$\theta_{TIR} = \pi, 2\pi, 3\pi, \ldots, n\pi$$
$$n = 1, 2, 3, \ldots,$$

Light entering the fiber at angle beyond the critical angle and at angle where $\theta_{TIR} = \pi, 2\pi, 3\pi, \ldots,$ under goes total internal reflection only without any part of the incident ray being lost inform of refraction. The signal travels through the fiber from source from one end to the target at the other end without attenuation.

Light entering the fiber at any angle other than, $\theta_i = \pi, 2\pi, 3\pi, \ldots,$ which is a prime condition for total internal reflection, will partially exit through the cladding by a phenomenon known as refraction.

Conclusion

Having extend Snell's law to a system where the angle of incidence of rf signal is greater than the critical angle for a pair of optical system, it was found that the angle of incidence greater than the critical angle is not a sufficient condition for total reflection but the angle of incidence must be an integral product of Pi, $\theta_{TIR} = \pi, 2\pi, 3\pi, \ldots,$

The findings of this study must be carefully considered when designing effective communication in optic fiber to avoid signal attenuation in form of refraction, dispersion and nonlinear effect. By understanding the principles of TIR and optimizing optical fiber design, system designers can improve the performance and reliability of optical fiber communication systems.

It is recommended that this study be carefully considered when designing optical fiber system.

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