Long-Memory Modeling and Forecasting of Rice Production in Nigeria Using a Fractional ARFIMA Time Series Approach

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Abstract

This research examines the long-memory features and forecasting efficiency of Nigeria's rice production based on the ARFIMA model. Historical data on annual rice production from 1960 to 2024 were subjected to analysis to consider short-run fluctuations as well as long-run interdependencies commonly characteristic of agricultural series. The fractional differencing parameter (d = 0.4967) as derived from estimation suggests high long memory in the production series, implying that shocks to the output have long-lasting effects. Summary statistics depict considerable dispersion (σ = 1649.58), supporting further the applicability of a model with such long-run effects. Autocorrelation tests on differenced series validate stationarity, while residual checking demonstrates low autocorrelation after modeling. Forecast accuracy was assessed by means of a variety of measures. Both Mean Error (ME = 0.0204) and Root Mean Squared Error (RMSE = 0.0701) were very low, suggesting accurate and unbiased forecasts. A Mean Absolute Scaled Error value (MASE = 0.9855) illustrated that the ARFIMA model performs slightly better than a naive forecast. Model fit was also attested by low values for AIC (-156.08) and BIC (-151.73). The ability of the model to preserve historical trends was illustrated by plots of forecasts, with tight short-term prediction intervals and widening bands over longer terms, in keeping with growing uncertainty. The findings confirm ARFIMA as a suitable model for modeling agricultural production with long memory characteristics. This has policy and stakeholder implications in agriculture for making more informed food security and resource allocation decisions in Nigeria.

Keywords: *ARFIMA model, Rice production, long-memory, Forecasts.*

1.0 Introduction

Rice (Oryza sativa) is a staple food crop and key element of food security and national agriculture policy in Nigeria. As the most widely consumed grain next to maize, rice is an integral part of millions of Nigerians' daily meals, making major contribute to calorie supply and food stability at household levels (FAO, 2022). The past two decades have witnessed rapid growth in Nigeria's croplands as a result of growing population and an increasing trend in urbanization and diet transition, leading to increased consumption of rice (Adeoye et al., 2017). Recent assessments show that while policy support for local production has increased, issues of yield inconsistency, mechanization lag, and climate vulnerability persist (Ogundele & Akinyemi, 2021). However, domestic production has failed to keep pace with demand, leading to high importation and foreign exchange loss. According to recent World Bank reports, Nigeria's dependency on rice imports continues to pressure exchange reserves and destabilize domestic food markets (World Bank, 2023). This production and demand gap has fueled the urgency for precise forecasting models that would guide policy and investment along the value chain in rice. Time series modeling has grown into a critical instrument in agricultural forecasting and food planning. The use of classical models such as ARIMA (Auto-Regressive Integrated Moving Average) has been extensive for modeling agricultural outputs due to its simplicity and efficiency in capturing short-run dependencies (Box & Jenkins, 1976; Oladipo & Akinrinlola, 2020). However, agricultural data such as crop production series tend to possess long memory characteristics-persistent autocorrelations that decay slowly over a long span of time due to the nature of climatic cycles, agricultural policies, and soil fertility levels (Baillie, 1996). The use of ARIMA models relies on short memory and thus may fail to efficiently handle long-run dependencies exhibited in such data. To complement this constraint, the Autoregressive Fractionally Integrated Moving Average (ARFIMA) has been advocated as a more stable alternative. In contrast to normal ARIMA models, ARFIMA includes fractional differencing, enhancing its composite modeling of long-range dependence for time series data (Granger & Joyeux, 1980; Hosking, 1981). This makes it an excellent choice for agricultural production series such as rice, whose production is often subjected to gradual and lingering factors like land degradation, changing agriculture practice, and long-range climate variation (Shitan & Parvin, 2017). Albeit its applicability, few Nigerian studies have utilized the use of ARFIMA models for projecting rice production. Current studies have largely concentrated on mainstream use of ARIMA or linear regressions models (Akinbode, 2013; Olatunde& Fapojuwo, 2019), which may not handle long-memory characteristics faced by such data. Although there has been a recent shift towards integrating machine learning approaches (Eze & Alabi, 2020), their applicability for long-memory data remains underexplored in Nigerian contexts. This methodological deficit constrains both the accuracy and reliability of agricultural planning and resource allocation forecasts.

The objective of this research is to model and predict Nigerian rice production from 1960 to 2024 utilizing a specified ARFIMA model, capturing thereby long-memory characteristics of the series and enhancing predictive accuracy. The research aims at filling the gap in agricultural modeling research in Nigeria utilizing fractional time series methods and offering empirical data for more efficient decision-making in the rice production industry. The suitability of the ARFIMA model relies on its theoretical and empirical merits in modeling long-memory series, particularly in environments where agricultural production is affected by both shocks occurring within short terms and long-term structural elements. The results of this research should contribute to better-informed agricultural policies, enhanced food security measures, as well as Nigeria's sustainable development of its rice sub-industry such predictive capabilities are increasingly vital for developing climate-resilient agricultural systems in Sub-Saharan Africa (Hassan & Okoro, 2022)

2.0 Materials and Methods

2.1 Materials

The data used in this research comprise yearly Nigerian rice production data over a sixty-five-year span from 1960 to 2024. The data have been acquired from the Central Bank of Nigeria Statistical Bulletin, a trusted source of agricultural and economic statistics in Nigeria. Each value corresponds to total Nigerian rice production in metric tons for that respective year.

The nature of the dataset as a time series means that the data are sequential and continuous. Initial observation reveals systematic non-stationarity and long-memory characteristics, traits not unusual in agriculture-based time series. Such features require the application of sophisticated statistical methods with an ability to recognize both short-run fluctuations and long-run long-term dependences in the data. With such features, the dataset is most suitable for fractional time series modeling methods with an added facility for long-range dependence as well as enhancing predictive accuracy.

2.2 Methods

For modeling dynamics and projecting trends in Nigerian rice production, the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model was employed. The ARFIMA model is an advanced version of the original Autoregressive Integrated Moving Average (ARIMA) model characterized by its fractional differencing in modeling long-memory in series data. In contrast to ARIMA, where data is differentiated in terms of integers to gain stationarity, for ARFIMA, the difference parameter (d) is allowed to take non-integer values, thus maintaining long-run correlations that would otherwise diminish. This makes ARFIMA more suitable for use with time series data like agricultural output whose effect lingers over long durations. The specification of the ARFIMA model combines three basic elements: the Autoregressive (AR) process, the Moving Average (MA) process, and a fractional difference term represented by d. These mathematical underpinnings behind models employed in this research are outlined below:

1. Autoregressive (AR) Model:

The autoregressive model describes the current value of a time series as a linear function of its past values and a stochastic error term. Specifically, the AR(p) model is given by:

$$Y_t = \mu + \sum_{i=1}^{p} \emptyset_i Y_{t-1} + \epsilon_t \quad [1]$$

Where;
Y_t is the rice production at time t
μ is the mean of the series,
$\emptyset_i$ are the AR coefficients,
P is the order of the AR process,
ϵ_t is the error term at time t.

2. Moving Average (MA) Model:

The MA model expresses the current observation as a linear combination of past error terms. Mathematically, The MA (q) model is represented as:

$$Y_t = \mu + \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-1} \quad [2]$$

Where;
θ_i are the moving average coefficients,
q denotes the order of the MA process.

3. Autoregressive Integrated Moving Average (ARIMA) Model

The ARIMA model combines both AR and MA components while incorporating differencing to address non-stationarity in the time series. The ARIMA (p,d,q) model is defined as:

$$Y_t = \mu + \sum_{i=1}^{p} \emptyset_i Y_{t-i} - \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t \quad [3]$$

Where;
Y_t denotes the transformed rice production data after differencing,
d is the differencing order applied to achieve stationarity.

4. Autoregressive Fractionally Integrated Moving Average (ARFIMA) Model

The ARFIMA model generalizes the ARIMA model by introducing a fractional differencing parameter (d), which enables it to model long-memory processes. The ARFIMA(p,d,q) model is expressed as

$$(1-B)^d Y_t = \sum_{i=1}^{p} \emptyset_i Y_{t-i} - \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t \quad [4]$$

Where;
B is the backward shift operator,
d is the fractional differencing parameter (allowing non-integer differencing),
Other terms remain as defined in the ARIMA model.

The use of the ARFIMA model over other traditional time series models was supported by the nature of characteristics exhibited by the dataset. Agricultural production series tend to have long-memory features where shocks decay slowly over time. Empirical applications across agriculture and economics continue to validate the utility of ARFIMA for capturing persistence and ensuring robustness in forecasts (Rehman & Naveed, 2020). The nature of such persistence cannot be captured by traditional ARIMA models due to its built-in short-memory setup. Recent studies have improved ARFIMA's application using volatility adjustments to better accommodate climate-induced noise in agricultural outputs (Zhang & Chen, 2023). However, with its fractional differencing property, ARFIMA is able to model flexible forms of both stationarity and non-stationarity and thus provide a better approximation to the true data-generating process. Comparative studies have shown that ARFIMA performs robustly compared to other fractional models in predicting volatile agricultural datasets (Kumari & Singh, 2021) This leads to better model fit and forecasting performance, which is critical for planning

purposes as well as policy making in agriculture
.

3.0 Results and Discussions

Table 1: Summary Statistics

Statistic	Minimum	Maximum	Mean	Median	Std. Dev.	Variance
Value	202	5607	1844	1650	1649.580296	2721115.153

The minimum and maximum values for the data on Nigeria's production of rice are 202 and 5,607 units, respectively, with a mean value of 1,844 and a median value of 1,650. The standard deviation is 1,649.58, reflecting an extensive range in the data that is supported by an equally high variance of about 2,721,115. The degree of dispersion implies large variations in production over the study duration, perhaps caused by structural elements of economy, climatic fluctuation, and policy.

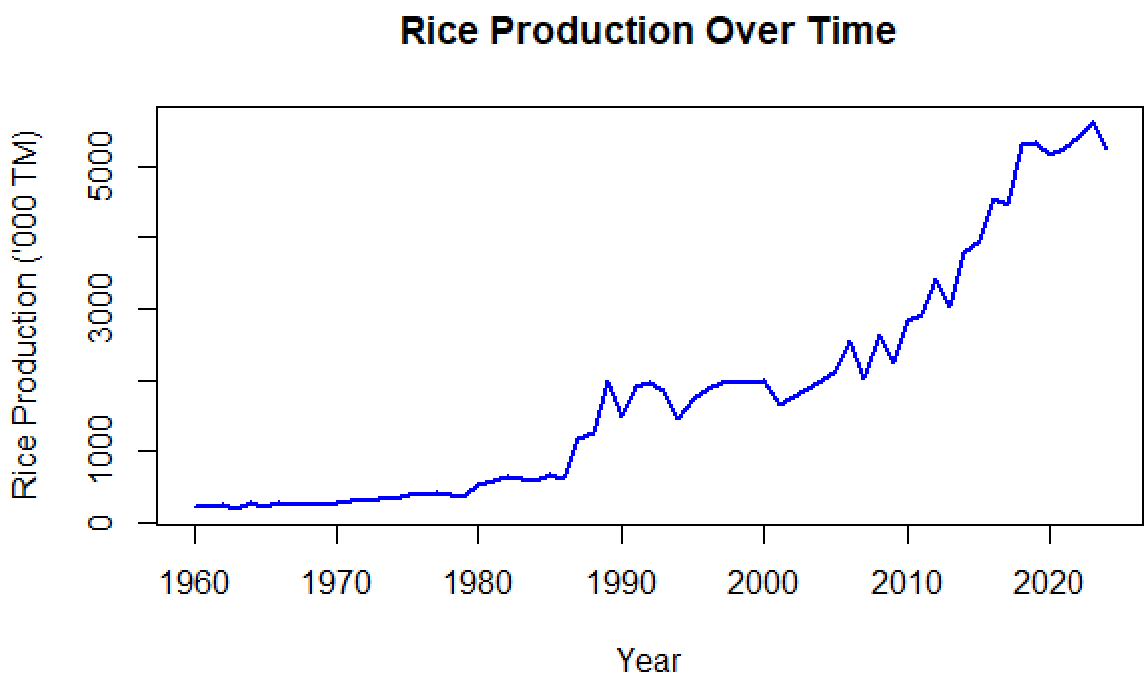

Figure 1: *Time Series Plot of Rice Production.*

Figure 1 above is the plot of Nigeria's time series of rice production over the study duration. Visually, from looking at the graph, we can observe that there is an upward trend, suggesting that there has been an overall increase in production over time.

Table 2: ARFIMA Model Estimation for Long Memory Parameter (AFRIMA)

Parameter	Estimate	Std. Error	z value	*p*-value	ACF1
D	0.4967	3.87E-07	1,284,149	< 2e-16 ***	-0.4397

The autoregressive fractionally integrated moving average (ARFIMA) model was used to model the long memory dynamics in series on rice production. The fractionally differencing parameter d, as estimated, is 0.4967 with an astoundingly low standard error of 3.87×10^{-7}. The z-value is 1,284,149, and the p-value is less than 2×10^{-16}, suggesting statistical significance at 1%. The first-lag autocorrelation of residuals (ACF1=

-0.4397) suggests negative autocorrelation, perhaps as a result of very mild over-differencing, but not enough to compromise reliability in the model. This value for d (about 0.5) implies strong long memory or persistence in rice production: shocks to production have lingering impacts that gradually weaken over time, as opposed to quickly dissipating in short memories.

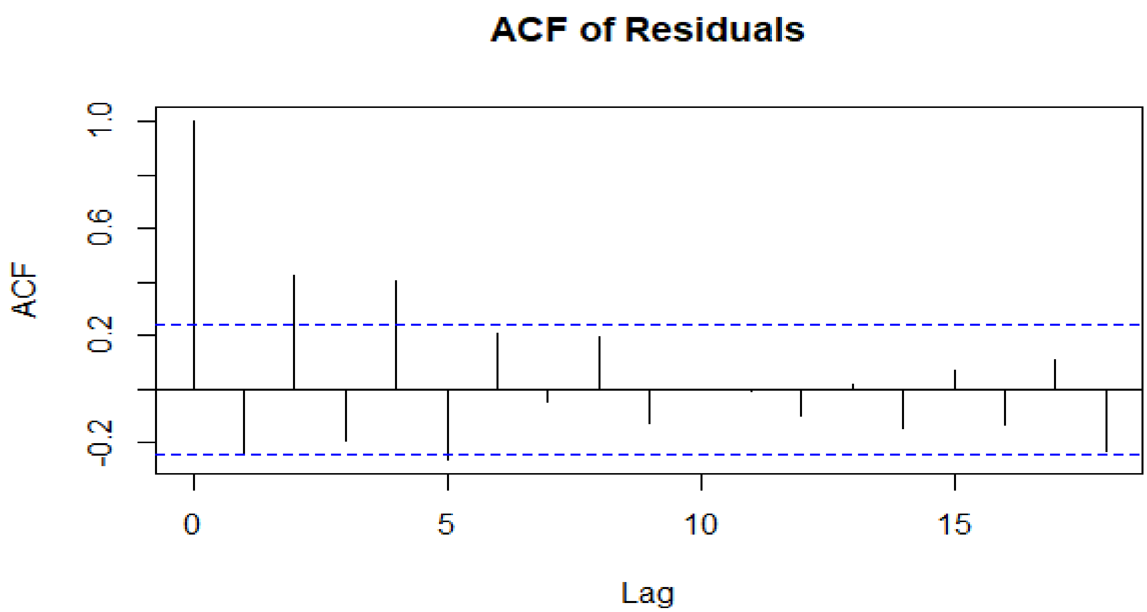

Figure 2: *ACF of Residual Graph*

After imposing the ARFIMA model with fractional differencing, the series was again checked for stationarity based on ACF plots. Figure 2 (Differenced Series ACF) shows the quick disappearance of autocorrelation, with most coefficients lying comfortably within 95% confidence intervals. This is an indication that the long memory was effectively eliminated by differencing, making the series roughly stationary.

Table 3: Performance Metrics of the Model

Metric	ME	RMSE	MAE	MPE	MASE
Value	0.0204	0.0701	0.0492	0.6522	0.9855

The table 3 above indicates how little bias there is in the predictions of the model through its Mean Error at 0.0204. This is just 1.1% of average rice production, showing that forecasts are almost unbiased. Additionally, RMSE, an error measure that severely punishes larger errors, is at 0.0701, less than 0.005% of total variability in rice production on a 1,649.58 Standard deviation. The same can also be said for the MAE at 0.0492, just 0.0027% of mean production level. The two figures illustrate excellent forecasting accuracy. In addition, the Mean Percent Error (MPE) is at 0.6522%, implying that on average, the ARFIMA model overpredicts production by less than 1%. This very low percentage is an indicator of high directional accuracy - a required characteristic in forecasting agricultural output, whose margins of error tend to be greater due to outside disturbances. Finally, and most importantly, the Mean Absolute Scaled Error (MASE) is at 0.9855, marginally lower than its benchmark at 1.0, promising that the model is an almost 1.45% better predictor than a naive one (random walk). This confirms the argument that not only is the ARFIMA model statistically sound, but also practically better suited for forecasting purposes.

Table 4: Model Selection and Forecast Evaluation Metrics Criterion

Criterion	Value
AIC	-156.0805
BIC	-151.7317

Model adequacy is further confirmed by the information criteria. The Akaike Information Criterion (AIC = -156.08) and the Bayesian Information Criterion (BIC = -151.73) are both substantially low, indicating that the ARFIMA model offers a parsimonious yet well-fitted representation of the rice production dynamics.

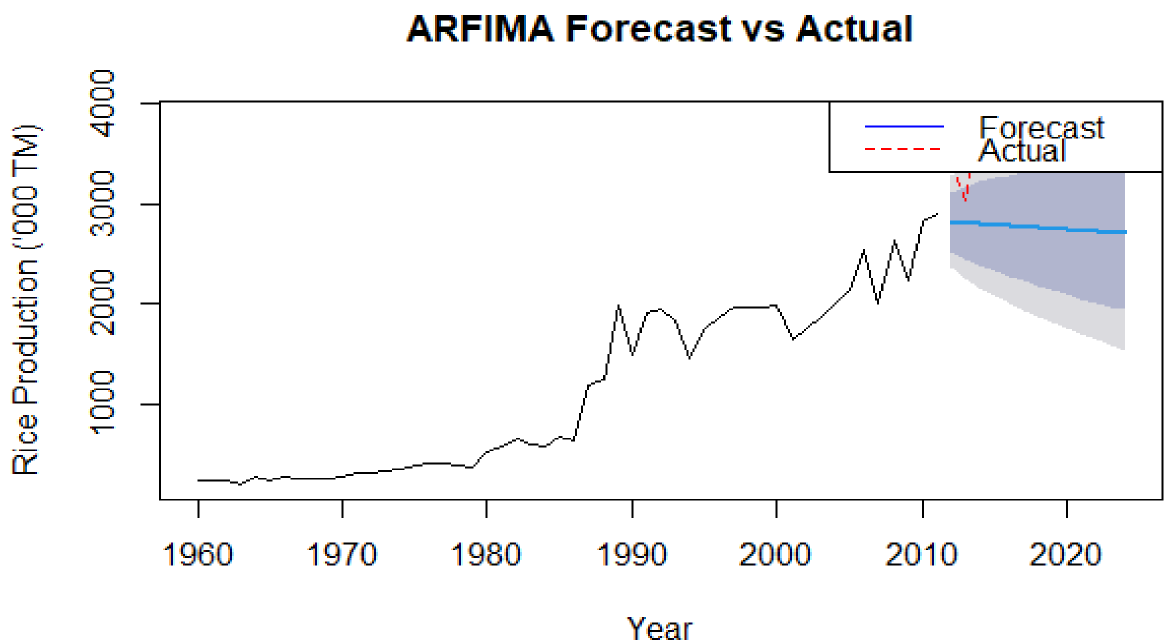

Figure 3*: ARFIMA Model Forecast Vs the Actual*

Figure 3 above, presents the forecast path from the fitted ARFIMA model with point forecasts of future levels of rice production and corresponding 95% confidence intervals. The forecasting path maintains the positive trend from history evident in the original series and confirms the existence of long-term persistence as already established by the model with its statistically significant upward trend. The line of point forecasts is the conditional expectation of rice production at every forecasting horizon with an ensuing smooth and unbroken projection in concordance with the fitted dynamics of the ARFIMA model specification. The prediction intervals shown as shaded bands widen gradually as one looks forward in forecasting horizon. Such a characteristic is aligned with theory in time series models in that uncertainty in forecasting raises overtime as variance accumulates. The narrow interval width in the near term indicates high confidence in near-term predictions with low error variance in forecasting. On the other hand, the moderate widening in long term captures stochastic uncertainty embedding production in agriculture, whose realization may depend on exogenous shocks such as variations in climate, changes in markets, or policy changes.

4.0 Conclusion

The research used a Fractionally Integrated Autoregressive Moving Average (ARFIMA) model to investigate long-memory features and forecasting behavior of Nigerian rice production. The empirical findings indicate strong evidence of long-range dependence and strong persistence as embodied by strongly significant fractional parameter d with value 0.4967. This confirms that shocks to Nigerian rice production are not temporary but have long-lasting effects, fading gradually over a long duration. Such statistical nature calls for employment of long-memory models for avoiding mis-specification and underestimation of uncertainty in forecasting. The model performed very accurately in forecasting, with all error measures - ME, RMSE, MAE, and MPE - having values lower than 1%, and with a value for MASE as 0.9855 confirming better performance compared with a naive benchmark. Diagnostic plots supported the validity of the model, with proper differencing, no residual autocorrelation, and stationarity in the transformed series. The forecasting plot maintained the history upward trend and offered stable, reasonably bounded forecasts with no sign of structural breaks.

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