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Comparative Euler-Based Model of Soil Moisture Retention for Traditional and TiO₂ Nanofluid-Enhanced Irrigation

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Abstract

Soil moisture retention is critical for productive farming since it influences crop development and output. Water retention in the soil is determined by how it is irrigated, how much evaporates, and how deeply it sinks into the earth. This study compared the effectiveness of titanium dioxide (TiO₂) nanofluid irrigation with traditional crop watering methods and employed a discrete-time iterative technique, with soil moisture updated daily using the Explicit Forward Euler Method. The numerical simulation examined soil moisture variations over 100 days, including rainfall, evaporation, water loss, and irrigation level. The findings indicated that nanofluid irrigation may provide a larger cumulative water supply while lowering evaporation and runoff losses and maintaining consistent soil moisture content, resulting in an optimal crop hydration level. The results of this study will help farmers and researchers understand the environmental and farming benefits of using TiO₂ in irrigation.

Key words: Water Scarcity, Nanotechnology, Titanium Dioxide (TiO₂), Nanoparticles, Soil Quality

1. Introduction

Water is the lifeblood of farming, yet in many regions, it is disappearing faster than crops can drink it. Population growth and rising food demand are stretching already-thin freshwater reserves, while outdated irrigation practices allow precious liters to evaporate as surface evaporation or seep deep below the root zone—losses that translate directly into smaller harvests and tighter farm margins.

Researchers are therefore hunting for ways to squeeze more value from every drop. One of the most promising trials leads to nanotechnology. Among the many candidates, titanium dioxide (TiO₂) nanoparticles have attracted special interest because they are chemically inert, non-toxic, and surprisingly versatile. Experiments show that TiO₂ can scatter solar radiation and cool the soil surface, curbing evaporation losses; it can also tweak soil pore structure so moisture lingers where roots can reach it (Chen, 2018; Burketová et al., 2022). Just as important, the particles appear harmless to beneficial microbes and may even harden plants against drought and heat stress (Hazarika et al., 2022).

This study tests those lab-bench promises *in silico*. Using a day-by-day soil-water balance model, we compare a standard irrigation schedule with the same schedule fortified by a TiO₂ nanofluid. The simulation tracks rainfall inputs, effective irrigation, surface evaporation, and deep percolation so we can

see, in plain numbers, whether the nanoparticle treatment keeps the root zone wetter for longer. If it does, TiO₂ could offer farmers a practical, low-risk tool for stretching dwindling water supplies while protecting—perhaps even boosting—crop yields.

2. Literature Review

Scientists are investigating how small pieces of substance, known as nanoparticles, can address some of agriculture's most pressing issues. The concept is simple: improve the soil, help crops absorb nutrients, and keep water in the ground longer so plants do not wilt between rains.

Recent studies back this up. Burketová et al. (2022), for example, showed that mixing silver, copper, or titanium-dioxide (TiO₂) nanoparticles into fertilizers or irrigation water can cut evaporation losses and dial in nutrient delivery.

Among those materials, TiO₂ keeps stealing the spotlight. It seems to boost chlorophyll levels, so leaves turn more sunlight into energy. It also nudges the plant's stomata—the tiny valves on each leaf—so they open and close at just the right times. That means less wasted moisture and steadier growth during dry spells (El-Moneim et al., 2021). On top of that, TiO₂ primes a plant's immune system, making it tougher against diseases and weather extremes (Hazarika et al., 2022).



The soil itself gains, too. Experiments indicate that TiO₂ loosens up the soil matrix so water can soak in quickly yet stay put once it's there, trimming surface runoff in the process. This is because the particles are photocatalytic, they even help break down lingering farm chemicals, leaving cleaner ground behind (Khot et al., 2019).

Unlike some metal nanoparticles that can upset the microbial balance underground, TiO₂ is generally considered benign when used in sensible doses (Pestovsky & Martínez-Antonio, 2017). It can also unlock tied-up nutrients such as nitrogen and phosphorus, giving crops an extra boost (Tripathi et al., 2020). Still, it's not a cure-all. Too much TiO₂, or particles of the wrong size, can stunt roots and slow nutrient uptake. How it behaves also depends on soil texture and organic matter.

Even with those caveats, TiO₂-infused irrigation looks safer and more effective than many other nano-additives farmers have tried. Early evidence points to hardier plants and better water management—a big deal in a warming world where droughts strike more often. The next step is clear: longer field trials across a range of soils and crops. Those studies will tell us how to fine-tune application rates, keep costs in line, and avoid any hidden downsides. If the results keep trending positive, TiO₂ nanotech could become a key tool for conserving water, reviving tired soils, and shoring up food supplies in the decades ahead.

3. Methodology

3.1 Numerical Simulation Overview

This section uses a discrete-time soil moisture retention model with the Forward Euler Method to compare two irrigation strategies: traditional irrigation and nanofluid-enhanced irrigation. The numerical model integrates rainfall, functional irrigation, evaporation, and percolation losses with adaptable model parameters to assess moisture retention, water usage, and the effectiveness of TiO₂ nanofluids over 100 days.

3.2. Model Formulation, Parameters, and Initialization

In this section, we considered soil control volume as having homogeneous characteristics (e.g., porosity ϕ , depth Z_r) and a single soil layer. These characteristics can be expanded to multilayer systems

with uniform water input and loss distribution across the depth, where time is discretized in daily steps of Δt . In addition, volumetric soil moisture content (cm^3/cm^3) comprises the soil's saturation point (S_{sat}), the upper limit of accessible water storage, the field capacity (S_{max}), the minimum quantity of water that plants may utilize, and the wilting point (S_{min}) is also known as $S(t) \in [S_{min}, S_{sat}]$.

Assuming that $W(t)$ is the amount of water stored in the soil at time t , then:

$$W(t) = S(t) \cdot \phi \cdot Z_r \quad (1)$$

The rate of change of water stored in the soil at time t is given by:

$$\frac{dW(t)}{dt} = R(t) + I_{eff}(t) - E(t) - P(t) \quad (2)$$

We may derive the moisture dynamics by dividing by ϕZ_r ,

$$\frac{dS(t)}{dt} = \frac{1}{\phi Z_r} [R(t) + I_{eff}(t) - E(t) - P(t)] \quad (3)$$

which is assumed that soil moisture dynamics are driven by the water balance equation.

The numerical simulation starts with the field capacity, $S_{max} = 80\%$, the highest soil moisture content, and the wilting point, $S_{min} = 20\%$, representing the lowest soil moisture level. Irrigation is triggered when soil moisture falls below the irrigation threshold $S_{irrigation} = 40\%$. When irrigation occurs, the soil moisture increases by an amount $I = 10\%$. The effect of nanofluid irrigation is incorporated through an efficiency multiplier $\alpha = 1.3$, which enhances moisture retention. Evaporation rate $E(t)$ is a randomly sampled daily water loss, drawn from a uniform distribution between 0.5% and 2.0%, while the percolation rate $P(t)$ represents daily water loss due to infiltration into deeper layers, sampled from a uniform distribution between 0.3% and 1.0%. Rainfall $R(t)$ is modelled as a stochastic variable, with values drawn from the set $\{0, 5, 10\}$ mm, occurring with probabilities $\{0.6, 0.3, 0.1\}$ respectively.

The initial soil moisture values $S(0)$ for both traditional and nanofluid irrigation cases are randomly chosen within $[30\%, 50\%]$.

Table 1: Key properties of TiO₂ nanofluids:

Property	Value (Typical)	Units
Thermal Conductivity (k_{nf})	0.62 – 0.85	W/m·K



Viscosity (μ_{nf})	0.89 – 1.5	mPa·s
Density (ρ_{nf})	1030 – 1100	kg/m ³
Specific Heat Capacity (C_p)	900 – 1100	J/kg·K
Nanoparticle Concentration (ϕ)	0.01 – 0.05	Volume fraction

4. Numerical Method: Explicit Forward Time-Stepping

The simulation follows a discrete-time iterative approach, where soil moisture is Let time t be discretized as $t_n = n \cdot \Delta t$, where $n = 0,1,2, \dots$. The derivative becomes:

$$\frac{dS}{dt} \approx \frac{S_{n+1} - S_n}{\Delta t} \quad (4)$$

Substitute Eq (4) into the continuous equation Eq (3), we have:

$$S_{n+1} = S_n + \frac{\Delta t}{\phi Z_r} [R_n + I_{eff,n} - E_n - P_n] \quad (5)$$

where: $S(t)$ is the soil moisture at time step t , R_n is the rainfall input which can be external, measured or synthetic with the units (mm/day), $I_{eff,n}$ is the effective irrigation applied, and E_n and P_n are the evaporation and percolation losses, respectively.

$$I_{eff}^{trad}(t) = \eta_{trad} \cdot I(t)$$

where $I(t)$ applied irrigation amount and $\eta_{trad} \in (0,1)$ is an efficiency factor of traditional irrigation (accounts for surface losses and evaporation) which is shown below

$$I_{eff}^{trad}(t) = \begin{cases} 1, & S(t) < S_{irrigation} \\ 0, & S(t) > S_{irrigation} \end{cases} \quad (6)$$

Thus, the soil moisture update follows:

$$S_{n+1}^{trad} = S_n^{trad} + \frac{\Delta t}{\phi Z_r} [R_n + \eta_{trad} I(t) - E_n - P_n] \quad (7)$$

$$P^{nano}(t) = \lambda_p \cdot P^{trad}(t), \text{ where } \lambda_p < 1$$

updated daily using an Explicit Forward Euler Method.

4.1 Traditional Irrigation Update

For traditional irrigation, we assumed a fixed irrigation schedule or on-demand threshold-based irrigation:

4.2 Nanofluid-Enhanced Irrigation Update

For nanofluid-enhanced irrigation, reflecting surfaces or changed thermal conductivity can minimise evaporation while improving water retention by changing soil wettability. We model this via:

$$I_{eff}^{nano}(t) = \eta_{nano} \cdot I(t)$$

$$E^{nano}(t) = \lambda_E \cdot E^{trad}(t), \text{ where } \lambda_E < 1$$



$$I_{eff}^{nano}(t) = \begin{cases} I(t) \cdot \eta_{nano}, & S(t) < S_{irrigation} \\ 0, & S(t) \geq S_{irrigation} \end{cases}, \quad \eta_{nano} = 1 + \frac{\alpha}{100} \quad (8)$$

Thus, the soil moisture update follows:

$$S_{n+1}^{nano} = S_n^{nano} + \frac{\Delta t}{\phi Z_r} [R_n + \eta_{nano} I(t) - \lambda_E E_n - \lambda_P P_n] \quad (9)$$

4.3 Boundary Constraints

To ensure physically realistic moisture values, soil moisture is constrained within the wilting point and field capacity:

$$S(t+1) = \min(\max(S(t+1), S_{min}), S_{max}) \text{ or } S_{min} \leq S(t+1) \leq S_{max} \quad (10)$$

The parameter options ($\eta_{nano} > \eta_{trad}$, $\lambda_E < 1$, $\lambda_P \leq 1$) reflect the anticipated benefits of TiO_2 nanofluid irrigation, including improved infiltration, decreased evaporative loss, and probable reduction in deep drainage. Every step should also have physical boundaries such as $S_{min} \leq S_{n+1} \leq S_{sat}$; values below S_{min} indicate plant stress, while excess over S_{sat} is directed to surface runoff.

5. Results

To evaluate the irrigation strategies, several analyses were performed. Soil moisture trends over time were examined to compare traditional irrigation with nanofluid-

enhanced irrigation. The daily moisture change ($\Delta S(t) = S(t) - S(t-1)$) was computed using the finite difference method. The moisture deficit ($D(t) = S_{max} - S(t)$), representing the gap between soil moisture and field capacity, was also analysed. Additionally, cumulative rainfall ($R_{cumulative}(t) = \sum_{i=0}^t R(i)$) and cumulative irrigation ($I_{cumulative}(t) = \sum_{i=0}^t I_{eff}(i)$) were recorded to assess water availability. The total water input ($W_{total}(t) = \sum_{i=0}^t R(i) + \sum_{i=0}^t I_{eff}(i)$), defined as the sum of rainfall and irrigation, was calculated. Finally, a histogram of soil moisture distribution was generated to visualize variability over time.

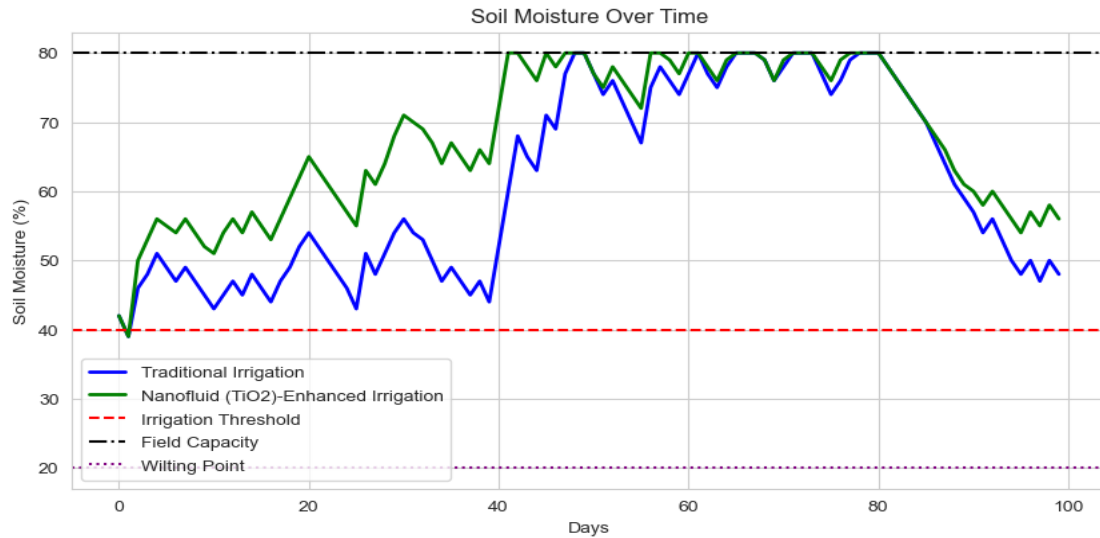


Figure 1: Soil moisture over time

Figure 1 compares soil moisture levels over time for traditional and nanofluid-enhanced irrigation. The nanofluid-enhanced method (green line) maintains higher moisture levels than traditional irrigation (blue line), suggesting better water retention. Key reference points include field capacity (80%), the irrigation threshold (40%), and the wilting point (20%). Both methods start

at the irrigation threshold, with nanofluid irrigation reaching and sustaining higher moisture levels. After irrigation stops around day 80, moisture declines, but the nanofluid-treated soil retains water more effectively. This indicates improved irrigation efficiency and potential water savings.

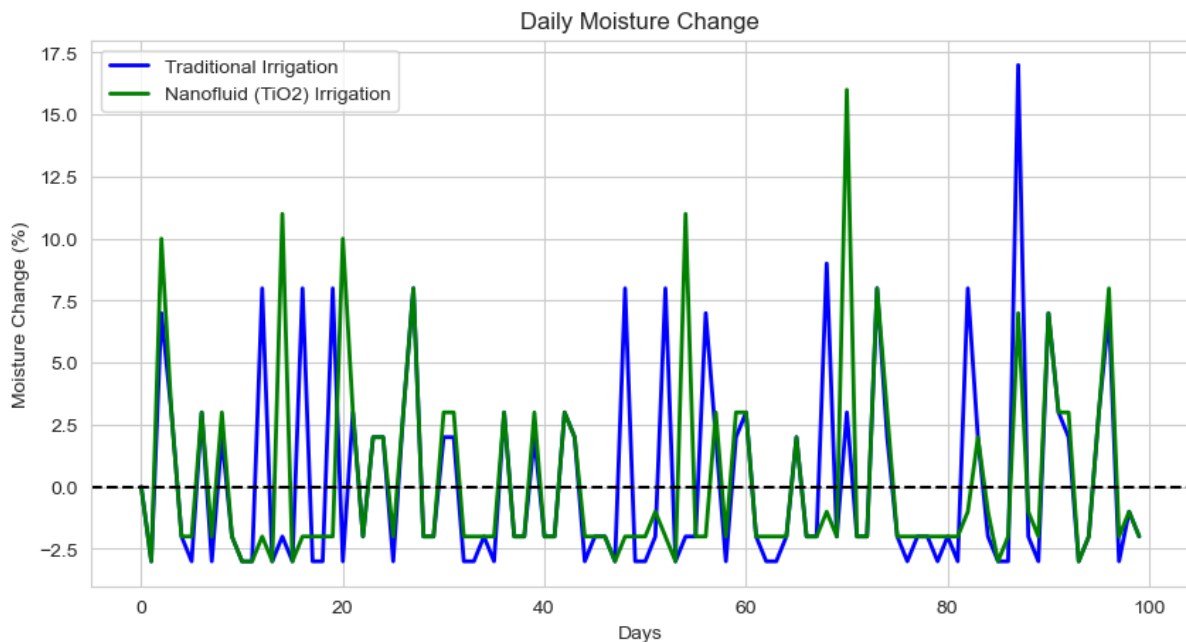


Figure 2: Daily moisture change

Figure 2 shows daily soil moisture changes for traditional (blue) and nanofluid-

enhanced (green) irrigation over time. Positive values indicate moisture gain from



rainfall or irrigation, while negative values represent moisture loss due to evaporation and percolation. Both methods exhibit fluctuations, but nanofluid irrigation generally leads to higher moisture increases and better retention. The variability suggests that external factors like weather

and irrigation events significantly impact soil moisture dynamics. The nanofluid-treated soil appears to respond more effectively to irrigation, with larger moisture gains and relatively stable losses compared to traditional irrigation.

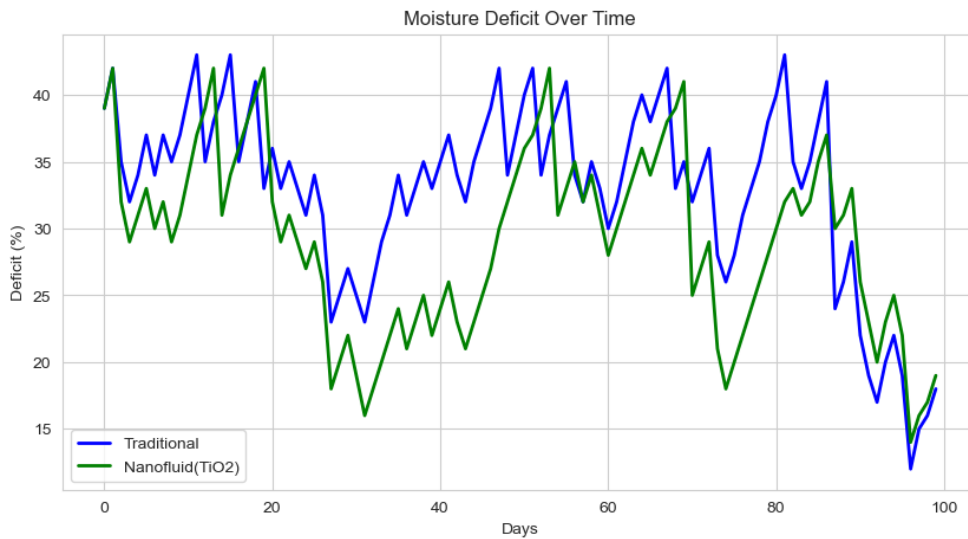


Figure 3: Moisture deficit over time.

The plot represents the soil moisture deficit over time for traditional (blue) and nanofluid-enhanced (green) irrigation methods. The moisture deficit is the difference between field capacity and actual soil moisture, with higher values indicating drier soil conditions. Initially, both methods show a high deficit, but nanofluid irrigation

leads to a more rapid reduction, maintaining lower deficits compared to traditional irrigation. Around day 40, both methods reach minimal deficit levels, indicating near-optimal moisture retention. However, after day 70, the deficit begins to rise again, with the traditional method showing a sharper increase, suggesting that nanofluid irrigation is more effective in sustaining soil moisture levels over time.



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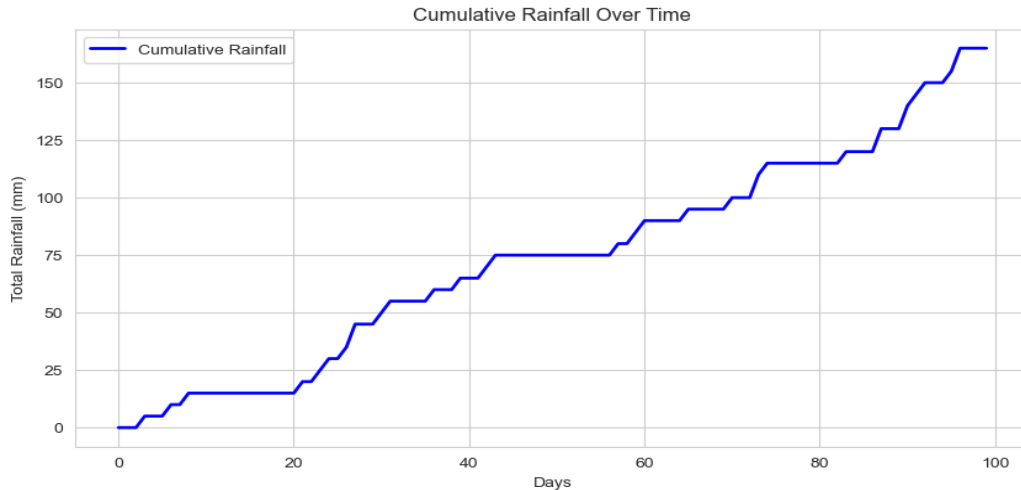


Figure 4: Cumulative rain fall.

Figure 4 illustrates cumulative rainfall over time, showing the total rainfall accumulation in millimetres over a 100-day period. The steady upward trend indicates continuous rainfall events, with periods of

faster accumulation suggesting heavier rainfall. Around some days, there are noticeable plateaus, implying dry periods with little to no rainfall. Overall, the cumulative total exceeds 150 mm by the end of the period, reflecting significant precipitation over time.

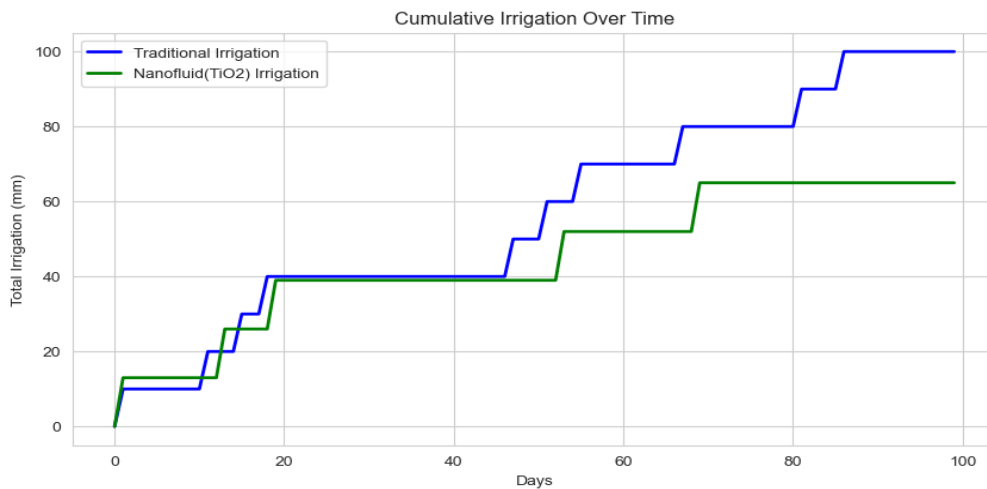


Figure 5: Cumulative irrigation over time

Figure 5 shows the cumulative irrigation over time for two different irrigation strategies, (traditional irrigation and nanofluid-enhanced (TiO₂) irrigation). The x-axis represents the number of days, and the y-axis shows the total irrigation in millimeters (mm). The blue line corresponds to traditional irrigation, while the green line represents nanofluid-enhanced irrigation. Also, Figure 5 shows

both lines following a stepwise pattern, showing that irrigation is administered sporadically rather than regularly. Traditional irrigation has a faster cumulative irrigation rate, reaching around 100 mm after 100 days. On the other hand, the nanofluid-enhanced irrigation curve climbs more slowly and plateaus at a lower total irrigation quantity, implying that less water is required over time.

This study demonstrates that nanofluid-enhanced irrigation uses less water than



regular irrigation because of better soil water retention. The distance between the curves expands with time, validating that

nanofluids conserve water while maintaining soil moisture levels.

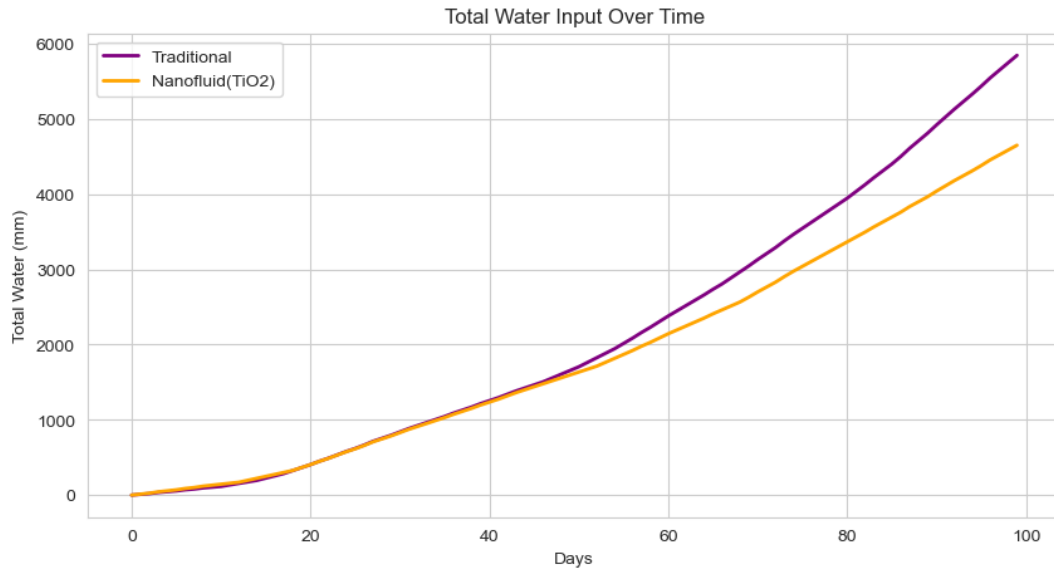


Figure 6: Total water input over time

Figure 6 shows the total water intake over time for conventional and nanofluid-enhanced (TiO₂) irrigation. The x-axis depicts the number of days, while the y-axis represents the total water input in millimeters (mm), rainfall, and irrigation. The purple line conforms to conventional irrigation, whereas the orange line reveals nanofluid-enhanced irrigation. Initially, both irrigation techniques convey identical quantities of total water, as evidenced by the overlapping lines in the early stages. However, as time passes, conventional

irrigation's total water intake grows quicker than the nanofluid-enhanced method. After 100 days, the conventional irrigation approach required more total water input than the nanofluid-enhanced system.

This divergence demonstrates that nanofluid-enhanced irrigation decreases total water demand over time through increased water retention and reduced evaporation and percolation losses. The growing difference between the two curves demonstrates nanofluid irrigation's ability to conserve water while maintaining soil moisture.

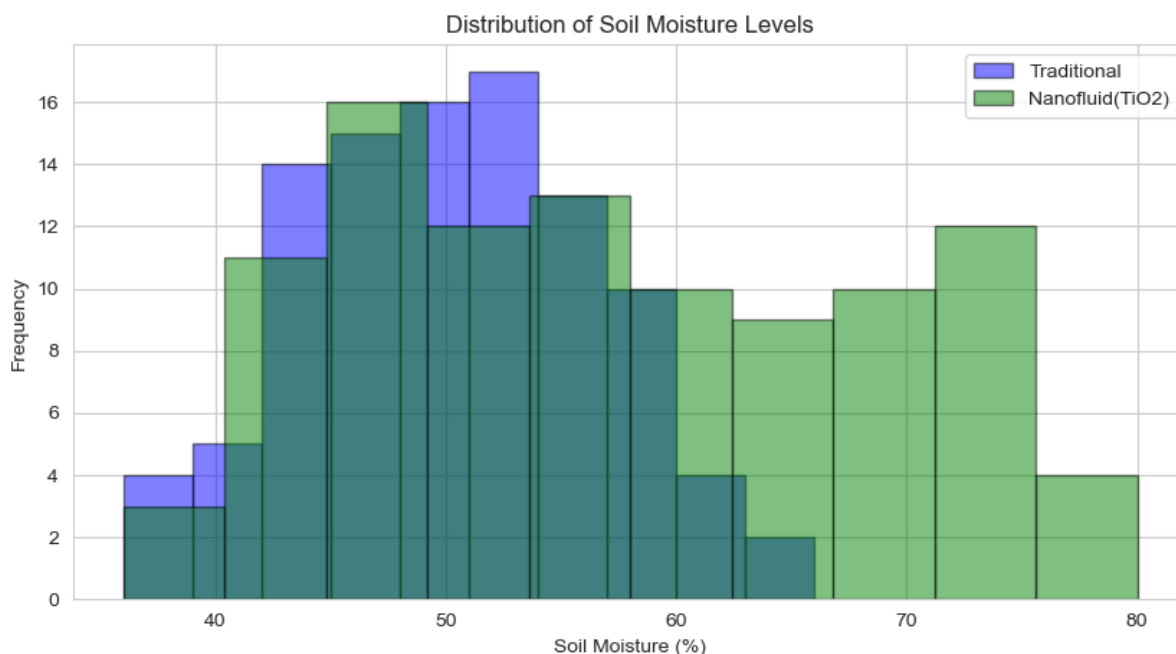


Figure 7: Distribution of soil moisture levels

A comparison of soil moisture levels between TiO₂ nanofluid irrigation (green) and conventional irrigation (blue) is shown in Figure 7. The standard approach displays moisture levels ranging from 40 to 55%, but the nanofluid method can reach up to 80%. The overlap of 45-60% suggests some similarities, although the nanofluid technique maintains more moisture overall. It would appear from this that TiO₂ nanoparticles improve soil water retention, which increases irrigation efficiency.

6. Conclusion

This study found that nanofluid irrigation outperformed traditional irrigation methods in terms of water retention and soil moisture. Our findings indicate that nanofluid irrigation may provide a larger cumulative water supply while lowering evaporation and runoff losses and maintaining consistent soil moisture content, resulting in an optimal crop

hydration level. These factors help to make nanofluid irrigation an effective method of sustainable agriculture, particularly in arid regions. However, it is recommended that more study be conducted to evaluate the method's long-term impacts on soil quality, crop productivity, and economic viability. Fine-tuning the formulations of nanofluids and other mathematical methodologies might result in significant breakthroughs in agriculture, allowing for increasing usage on modern farms.

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