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# Mathematical Modelling of the Moisture Retaining Characteristic of Clay Soil Over Some Other Soil Types

**Olaleye Olalekan A.✉, Olalude Gbenga A., Alabi Peter A., Olatidoye Waheed Y. & Alade Abigail F.**
Department of Statistics, The Federal Polytechnic, P.M.B. 231, Ede, Osun State, Nigeria
✉: hezekiaholaleye@gmail.com

**ABSTRACT**
The characteristic of clay soil to retain moisture better than some other soil types was modelled. Relevant assumptions were made based on the conditions under which the flow occurs. Using the usual Boussinesq approximation guided by these assumptions, the equations governing heat and mass flow were derived. The energy and momentum equations were both reduced to non-dimensional form and then to ordinary differential equations. These were subsequently resolved analytically, with numerical results computed using MATLAB software. The results were visually presented in graphs to illustrate the rate at which fluids passed through selected soil samples. Therefore, the model confirms the slower rate of fluid flow through clay soil compared to other soil samples considered.
**Keywords**: Boussinesq's approximation, clay soil, energy equation, fluid-flow, momentum equation, ordinary differential equation.

## 1.0 INTRODUCTION

Year after year, there has been a focus on how the planet can be a better place to live. This is the centre of the research world, trying to improve the life of man on earth. Thus, in various fields of study, ranging from applied sciences involving experiments, to Agriculture, to Engineering and Technology, and so on, much work has been done to improve the existing ones. Models are being developed to better illustrate different situations and to improve them. Some of these models are done in the area of heat transfer, which is involved and part of our day-to-day basic activities. For instance, Hud *et al* (2019) used a soil surface heating system to model the heat transfer mathematically, in a greenhouse. Their work revealed that the air distribution has a noteworthy effect on the temperature, which results in the optimum temperature at the greenhouse centre. Based on the distance, their analysis also established the fact that the soil's temperature under shelter reduces from its middle to the edge by around 2 to 6ºC.

Olaleye *et al* (2020) examined the effects that some physical features have on the movement of fluid through sand (as a porous medium). Their results showed that all the physical parameters considered, except for the Prandtl number all increased the flow of the fluid through the soil.

Rajesh *et al* (2013) worked on the heat transfer and slip flow (partial) over a stretched sheet in Nanofluid. The slip parameter and volume fraction of the nanoparticles, which emerged as the physical parameters after the principal equations had been resolved numerically, were examined by them on the speed and temperature report alongside skin friction.

Few others include Reddy *et al* (2011) that with Soret and variable suction, studied unsteady Magnetohydrodynamics free convective mass transport flow through infinite vertical permeable plate, Animasahun and Oyem (2014) which examined impact of "Soret, Variable Viscosity, Thermal Conductivity and Dufor on free convective heat and mass transfer of non-Darcian flow pass porous flat surface", Alabison *et al.* (2019) who studied the control of heat generation & radiation on Convective flow via an absorbent medium using boundary condition that is periodic in term of temperature, etc.

Given the above, this work mathematically models the moisture-retaining characteristic of clay soil over some other soil types.

## 2.0 MATHEMATICAL ANALYSIS

The flow involves two dimensional heat transfer via porous medium. The flow is also taken to be infinite toward the horizontal axis and $z'-axis$ (vertical axis) normal to it. The soil types used are considered to be porous media and also optically thin. The radiation in the direction of gravity is coming from the solar source. With Boussinesq approximation (1877), applying above assumptions, the principal equations became:

***Continuity equation***

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

***Momentum Equation***

$$\frac{\partial v'}{\partial t'} + w'\frac{\partial v'}{\partial z'} = \vartheta\frac{\partial^2 v'}{\partial z'^2} + g\beta(T' - T'_\infty) - \frac{w}{K'}\varphi v' \quad (2)$$

***Energy equation***

$$\frac{\partial T'}{\partial t'} + w'\frac{\partial T'}{\partial z'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial z'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial z} + \frac{Q_0(T' - T'_\infty)}{\rho C_p} \quad (3)$$

Subject to:

$v'(z,t) = V'_P + \eta'\cos(\omega' t')$, $T' = T'_w + \lambda'\cos(\omega' t')$ at soil's surface (i.e. $z = 0$) (4)

$v'(z,t) \to V'_\infty$ , $T' \to T'_\infty$ as soil's depth increases for an indefinite period (i.e. $z' \to \infty$) (5)

where, $z'$ being the soil's depth in dimensional type & perpendicular to $y'$. $t'$ & $w'$ are time and suction velocity correspond. $T'$, $T'_w$ and $T'_\infty$ are temperature, wall temperature & free stream temperature correspondingly. K, $\varphi$, $\rho$, $C_p$, $k$ & $q'_r$ are the permeability, porosity, density, heat capacity, thermal conductivity & heat flux correspondingly.

Using the dimensionless parameters,

$$t = \frac{t' w_0^2}{w}, \quad z = \frac{w_0 z'}{w}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \omega = \frac{w\omega'}{w_0^2}, \qquad v' = vV \quad (6)$$

The suction velocity as used by Mohammed (2013) was constant. But this can also be taken to be varying with time to make it more practicable. Thus, it is given as

$$w' = -w_0(1 + \varepsilon A e^{i\omega' t'}) \quad (7)$$

Where $w_0$ , A & $\omega$ are suction velocity (initial), suction parameter & oscillation frequency correspondingly.

In addition, the heat flux is written as:

$$\frac{\partial q'_r}{\partial z'} = 4\tau^2(T' - T'_\infty) \quad (8)$$

$\tau$ being the absorption coefficient.

Also, according to Akinpelu (2016), the time-dependent thermal conductivity is specified as

$$k = k_0(1 + \lambda t) \quad (9)$$

$\lambda$, $t$ & $k_0$ are parameters of thermal conductivity (variable), time & constant thermal conductivity.

Equation (2) then becomes:

$$\frac{\partial v}{\partial t} - (1 + \varepsilon A e^{i\omega t})\frac{\partial v}{\partial z} = \frac{\partial^2 v}{\partial z^2} + Gr\theta - \frac{v}{K}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t})\frac{\partial \theta}{\partial z} = \frac{1}{P_r}\left\{(1 + \lambda t)\frac{\partial^2 \theta}{\partial z^2}\right\} - R^2 + Q\theta \quad (10)$$

Subject to:

$$v = 1 + h_0\cos(\omega t) \;, \theta = 1 \text{ at } z = 0$$

$$v \to 0 \;, \theta \to 0 \text{ as } z \to \infty \quad (11)$$

Where the Prandtl number is given as,

$$P_r = \frac{w\rho C_p}{k_0}$$

the thermal Grashof number is,

$$Gr = \frac{wg\beta(T_w^* - T_\infty^*)}{W_0^2 V_\infty^*}$$

the permeability of the soil type is,

$$K = \frac{K^* W_0^2}{w^2 \varphi}$$

$$R^2 = \frac{4\alpha^2 \theta w}{w_0^2}$$ (the radiation parameter), and

$$Q = \frac{Q_0 w}{\rho C_p w_0^2}$$ (internal heat)

## 2.1 Method of Solution

Method of regular perturbation was deployed to reduce equations (8) and (9) into ordinary differential equations. This is done by assuming the solutions to be:

$$v(z,t) = v_0(z) + \varepsilon e^{i\omega t} v_1(z) + \cdots \tag{12}$$

$$\theta(z,t) = \theta_0(z) + \varepsilon e^{i\omega t} \theta_1(z) + \cdots \tag{13}$$

Higher order terms $o(\varepsilon)^2$ in equations (12) and (13) were neglected. After which the equations with their derivatives were put into equations (8) and (9), and then simplified further,

$$v_0'' + v_0' - \left(\frac{1}{K}\right) v_0 = -Gr\theta_0 \tag{14}$$

$$v_1'' + v_1' - \left(\frac{1}{K} + i\omega\right) v_1 = -Gr\theta_1 \tag{15}$$

$$(1+\lambda t)\theta_0'' + P_r\theta_0' + P_r Q\theta_0 = P_r R^2 \tag{16}$$

$$(1+\lambda t)\theta_1'' + P_r\theta_1' + P_r(Q - i\omega)\theta_1 = -P_r A\theta_0 \tag{17}$$

primes in the equations signify ordinary differentiation done with deference to z.
Solving equations (14) – (17) under the boundary conditions (18) and (19), the velocity and temperature distributions become:

$$\theta = C_1 e^{m_1 z} + C_2 e^{m_2 z} + C_3 + \varepsilon e^{i\omega t}(C_4 e^{m_3 z} + C_5 e^{m_4 z} + C_6 e^{m_1 z} + C_7 e^{m_2 z}) \tag{18}$$

$$\begin{aligned} v &= C_8 e^{m_5 z} + C_9 e^{m_6 z} + C_{10} e^{m_1 z} + C_{11} e^{m_2 z} + C_{12} \\ &\quad + \varepsilon e^{i\omega t}(C_{13} e^{m_7 z} + C_{14} e^{m_8 z} + C_{15} e^{m_3 z} + C_{16} e^{m_4 z} + C_{12} e^{m_1 z} + C_{18} e^{m_2 z}) \end{aligned} \tag{19}$$

Where,

$$m_1 = -\frac{P_r}{2(1+\lambda t)} + \sqrt{\frac{P_r^2}{4(1+\lambda t)^2} - \frac{P_r Q}{1+\lambda t}}$$

$$m_2 = -\left(\frac{P_r}{2(1+\lambda t)} + \sqrt{\frac{P_r^2}{4(1+\lambda t)^2} - \frac{P_r Q}{1+\lambda t}}\right)$$

$$m_3 = -\frac{P_r}{2(1+\lambda t)} + \sqrt{\frac{P_r^2}{4(1+\lambda t)^2} - \frac{P_r Q}{1+\lambda t} + \frac{P_r i\omega}{1+\lambda t}}$$

$$m_4 = -\left(\frac{P_r}{2(1+\lambda t)} + \sqrt{\frac{P_r^2}{4(1+\lambda t)^2} - \frac{P_r Q}{1+\lambda t} + \frac{P_r i\omega}{1+\lambda t}}\right)$$

$$m_5 = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{K}} \qquad m_6 = -\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{K}}\right)$$

$$m_7 = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{K} + i\omega} \qquad m_8 = -\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{K} + i\omega}\right)$$

$$C_1 = -C_3 e^{-m_1 z} \qquad C_2 = 1 + C_3\left(e^{-m_1 z} - 1\right) \qquad C_3 = R^2 / Q$$

$$C_4 = \frac{-C_6 e^{m_1 z}}{e^{m_3 z}} \qquad C_5 = -(C_4 + C_6 + C_7)$$

$$C_6 = \frac{-P_r A m_1 C_1}{m_1^2 + stm_1^2 + P_r m_1 + P_r Q - P_r i\omega} \qquad C_7 = \frac{-P_r A m_2 C_2}{m_2^2 + stm_2^2 + P_r m_2 + P_r Q - P_r i\omega}$$

$$C_8 = 1 + C_{10} e^{m_1 z} - C_{12} \qquad C_9 = 1 + \cos\eta - (C_8 + C_{10} + C_{11} + C_{12})$$

$$C_{10} = \frac{-GrC_1}{m_1^2 + m_1 - \frac{1}{K}} \qquad C_{11} = \frac{-GrC_2}{m_2^2 + m_2 - \frac{1}{K}} \qquad C_{12} = GrC_3 K$$

$$C_{13} = \frac{-C_{15} e^{m_3 z} - C_{17} e^{m_1 z}}{e^{m_7 z}} \qquad C_{14} = -(C_{13} + C_{15} + C_{16} + + C_{17} + C_{18})$$

$$C_{15} = \frac{-GrC_4}{m_3^2 + m_3 - \left(\frac{1}{K} + i\omega\right)} \qquad C_{16} = \frac{-GrC_5}{m_4^2 + m_4 - \left(\frac{1}{K} + i\omega\right)}$$

$$C_{17} = \frac{-GrC_6}{m_1^2 + m_1 - \left(\frac{1}{K} + i\omega\right)} \qquad C_{18} = \frac{-GrC_7}{m_2^2 + m_2 - \left(\frac{1}{K} + i\omega\right)}$$

## 3.0 RESULTS AND DISCUSSION

Transient temperature numerical results were computed and displayed on graphs. This was done by substituting the default values as given below into equation (18) which represent the transient temperature. Then, solar radiation stricture, internal heat generation (Q), as well as Prandtl number (Pr) was studied on the Temperature gradient of both the unsaturated and saturated sandy.

Table 1 displays thermal conductivities values for unsaturated and saturated sands.

*Table 1: Thermophysical properties of saturated soil samples in use*

| Soil | Permeability | Thermal Conductivity (W/m K) |
|---|---|---|
| Clay | 0.50 ($\times 10^{-1}$) | 0.64 |
| Sand | 5.00 | 0.44 |
| Loam | 1.00 | 0.52 |

Gary (2015)

Except stated otherwise, the below default values as used in some existing literatures which include Animasahun and Oyem (2014) among host of others were incorporated.

$P_r = 0.71$ , $Q = 0.01$ , $\omega = \pi/2$ , $\varepsilon = 0.01$ , $t = 1.0$ , $A = 0.5$ , $R = 0.1$ , $A_0 = 1.0$ ,

$h_0 = 1.0$ , $Gr = 2.0$

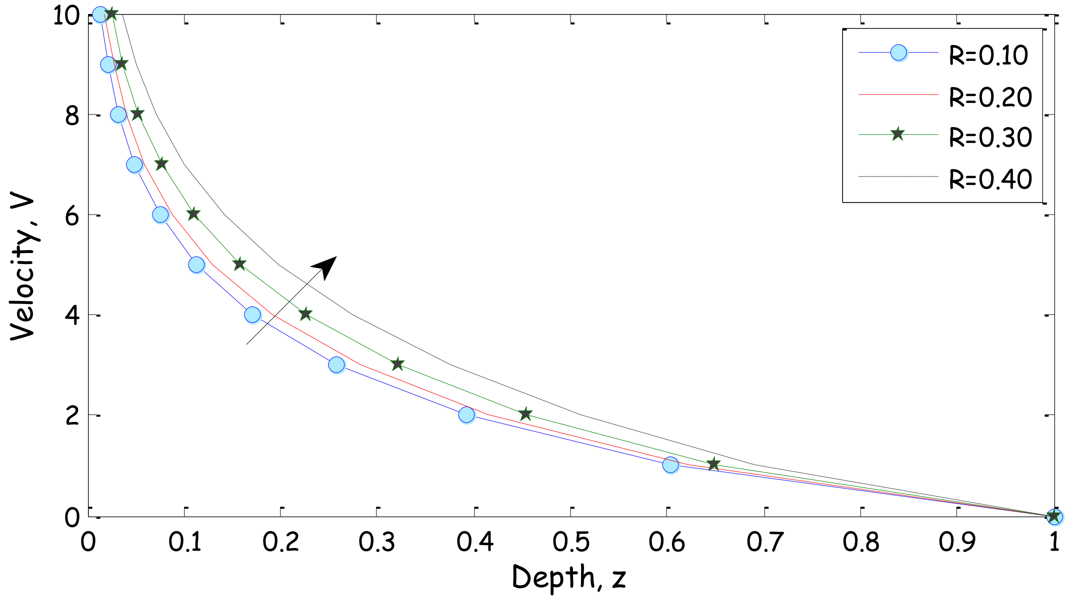


Figure 1: Effect of solar radiation on the fluid flow through clay soil

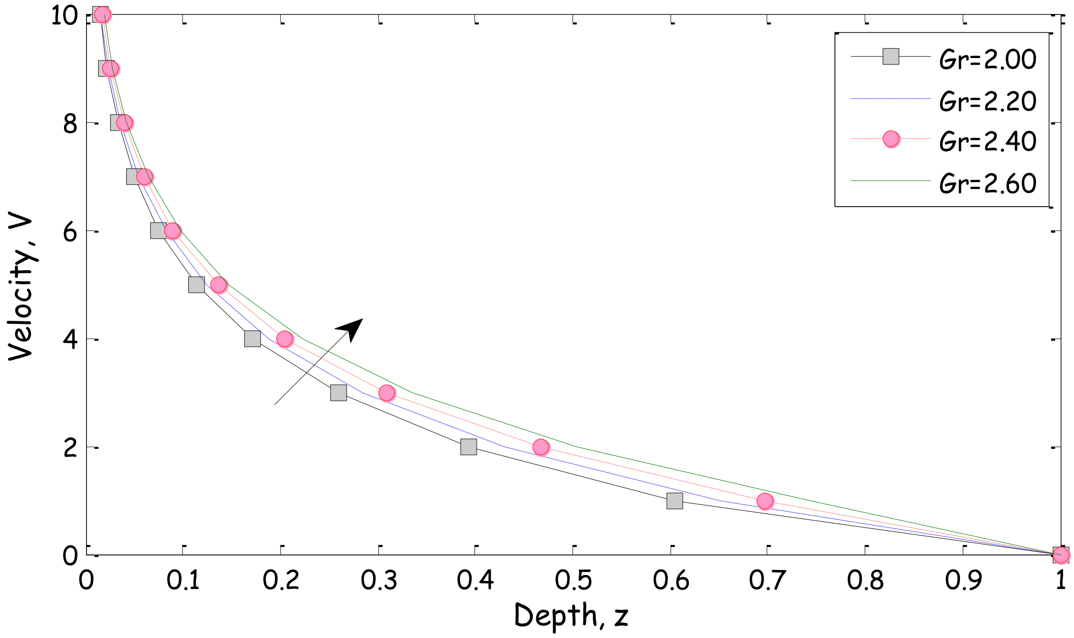


Figure 2: Outcome of Gr on fluid flow through clay soil

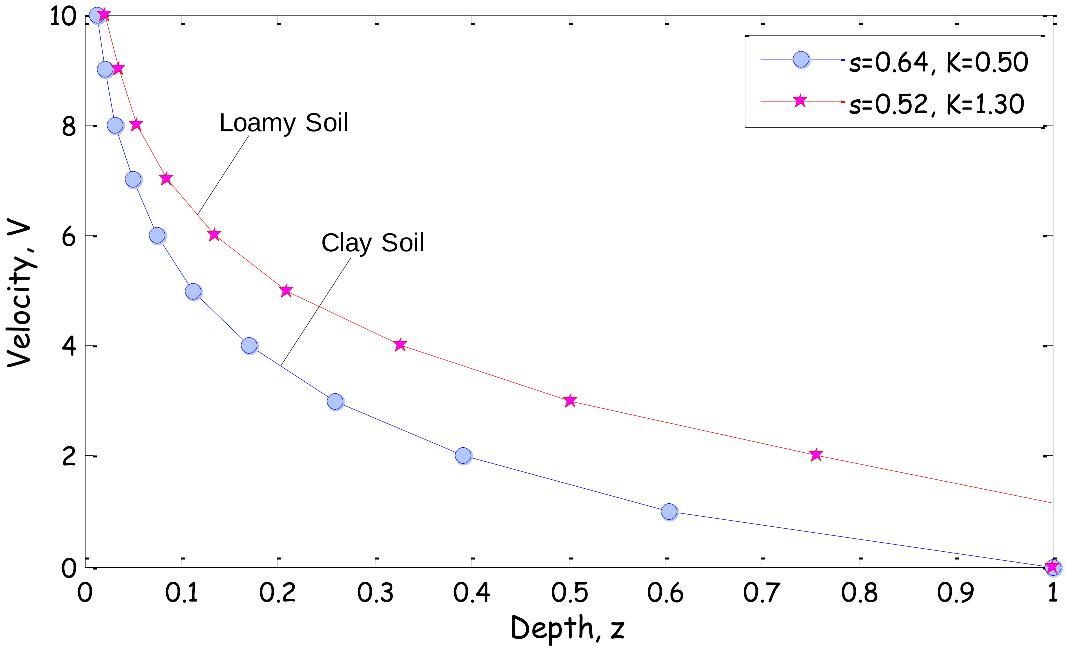


Figure 3: Velocity profile of the fluid flow through clayed and Loamy soils

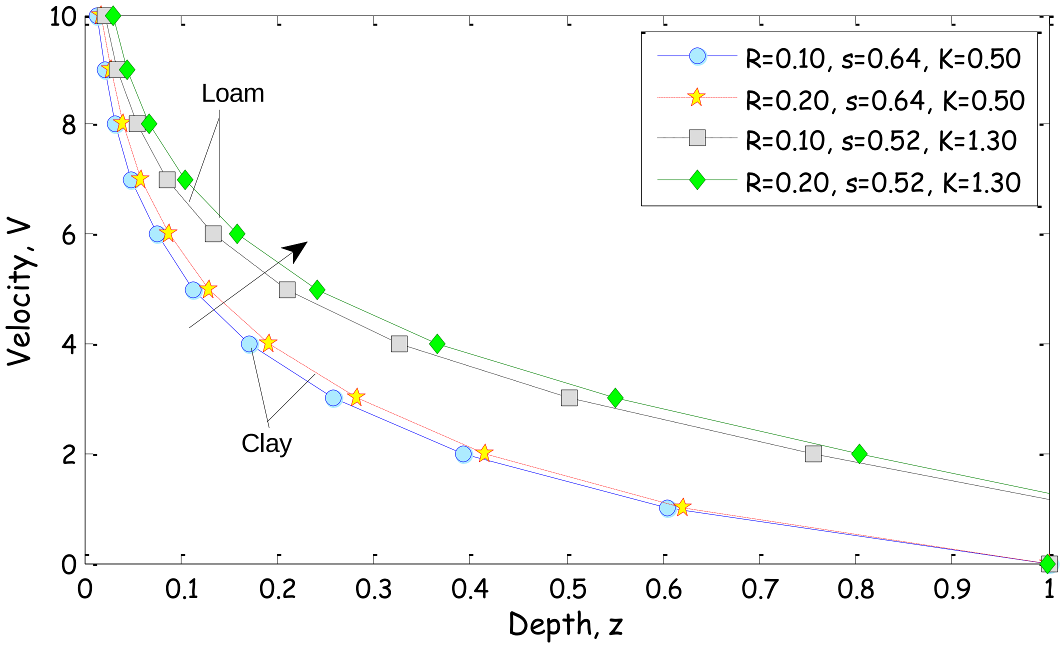


Figure 4: Velocity profile of effect of radiation on the fluid flow through clayed and Loamy soils

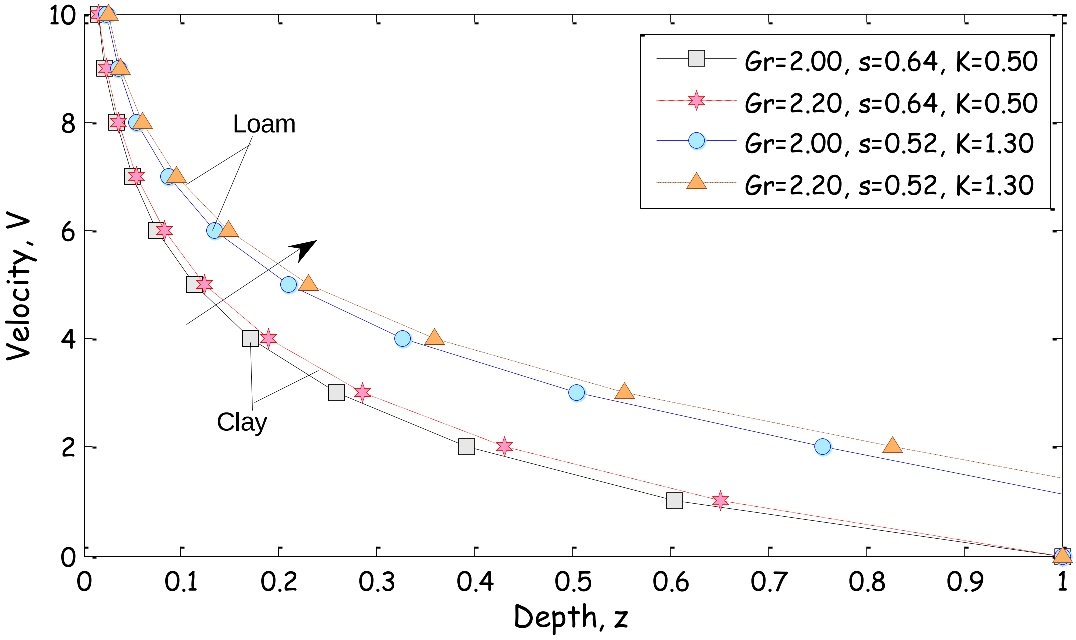


Figure 5: Velocity profile of control of Gr on fluid flow through clayed and Loamy soils

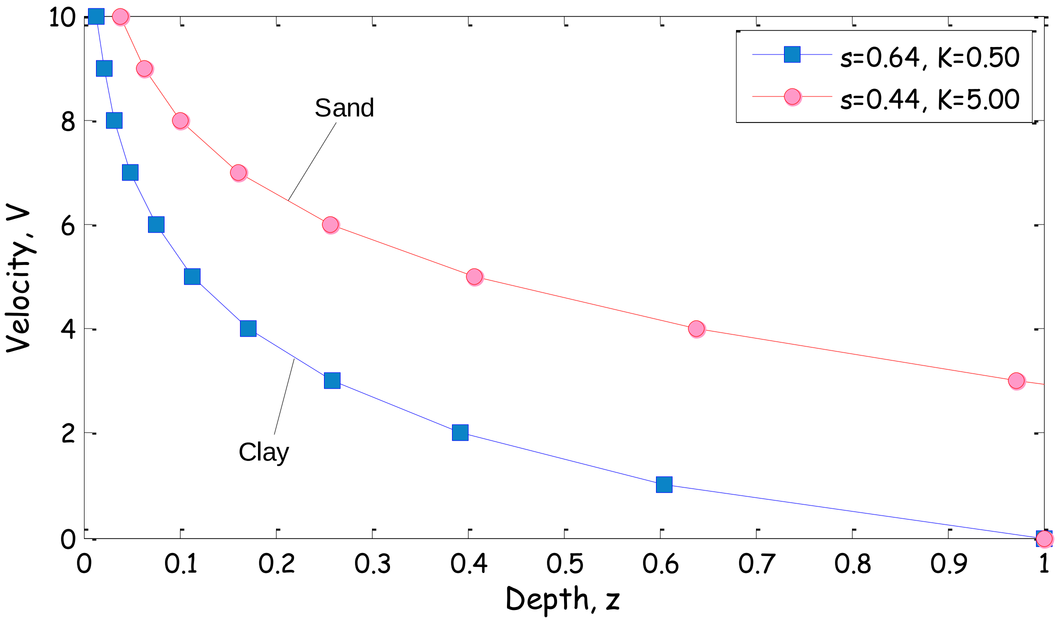


Figure 6: Velocity profile of the fluid flow through clayed and Sandy soils

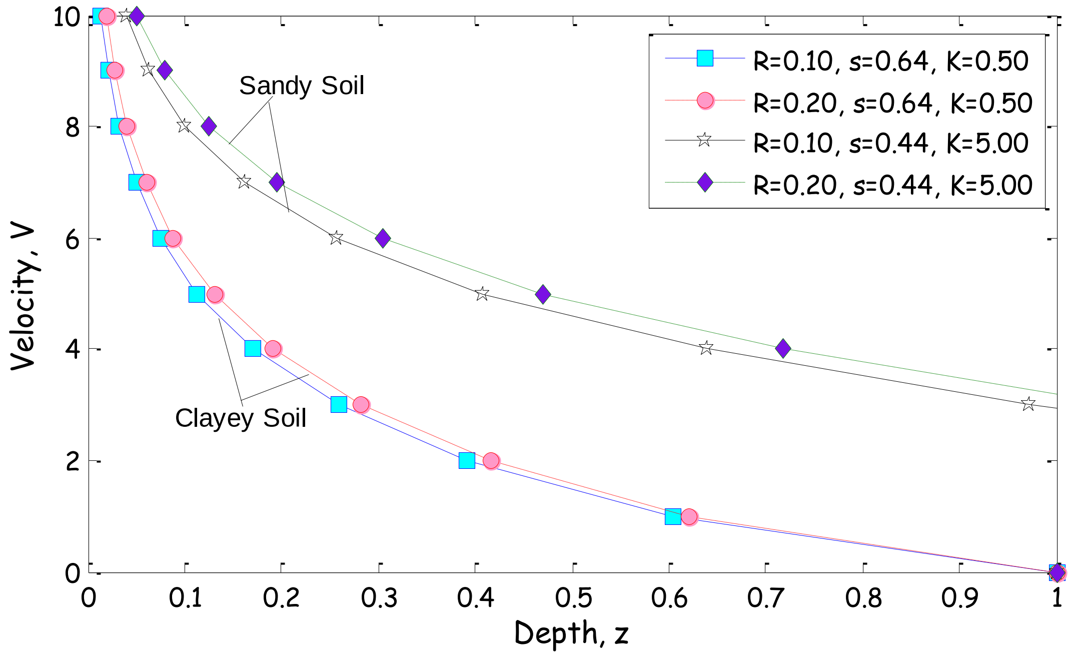


Figure 7: Velocity profile of the effect of radiation on the fluid flow through clayed and sandy soils

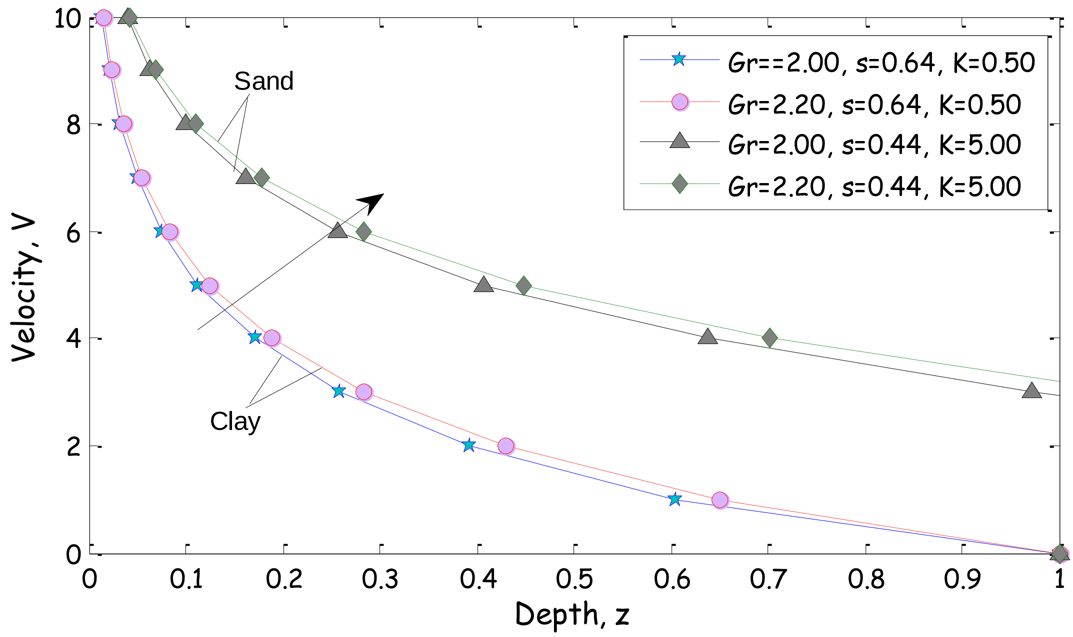


Figure 8: Velocity profile of effect of thermal Grashof number on the fluid flow through clayed and sandy soils

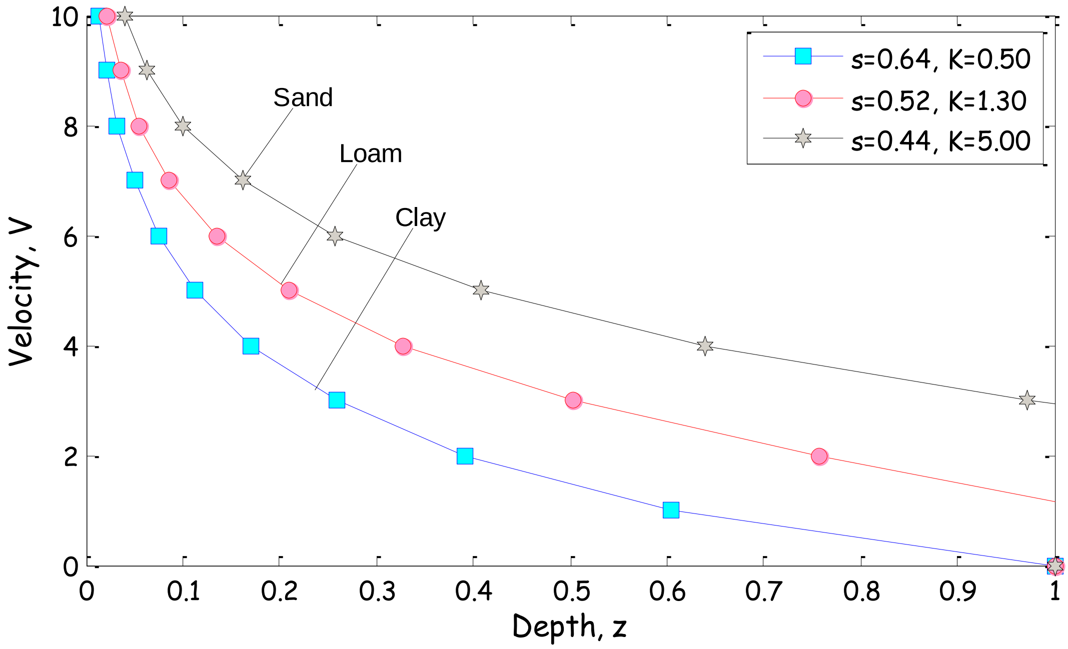


Figure 9: Velocity profile of the fluid flow through clayed, Loamy and Sandy soils

**4.0 CONCLUSION**

In figure 1 and 2, the effect of solar radiation and the thermal bouyancy force respectively was examined on the fluid flow through clay soil. It was discovered that both parameters incresed the fluid flow. Moreover, in figure 3, the flow rate of the fluid is more through the loamy soil than that of the clay soil when all other parameters are kept constant. However, figures 4 and 5 revealed that when both the radiation and bouyancy force increased, the flow rate and the level of the increase in the flow were almost the same. Furthermore, similar thing occurred when comparing the fluid flow through the clay soil and the sandy soil (figures 6, 7 and 8). Figure 9 displayed the rate of fluid flow through the three soils. It was evident that the rate was most through sandy soil, followed that loam and slowest through clay.

In conclusion, the characteristic of a clay soil to retain moisture over some other soil types was modeled in the studied. It was evident that an increasing solar radiation increases the velocity of fluid flow through the soils. Also, an increasing thermal buoyancy force increases the velocity of fluid flow through these soils. The rate of flow of fluid through clay soil however is lower than that of the other soils in comparison.

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