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Experimental

AN EXPONENTIAL-PARETO DISTRIBUTION APPROACH TO IMPROVING RAW MATERIAL QUALITY

Idowu, A.O. & Ajibode, I.A.

Department of Mathematics and Statistics, Federal Polytechnic Ilaro, Ogun State, Nigeria

ilesanmi.ajibode@federalpolyilaro.edu.ng

Abstract

This research investigates the use of the exponential-pareto distribution to improve raw material quality in cement production. Experienced researchers researched on the distribution and came up with the generalised form of the distribution. The weight of the raw components, aluminium, calcium, gypsum, iron, and silicon, was collected from 2007 to 2017. Using exponential pareto, average run lengths (ARL), control limit intervals (CLI), and process capability (CP) were calculated, and control charts were created. The investigation found that the control charts were statistically controlled, indicating that the distribution is effective. It was suggested that an exponential-pareto chart be used to control the quality of raw materials used in cement manufacture.

Keywords: Cement, distribution, exponential, pareto, production, raw material

1.0 INTRODUCTION

Control systems are used in industrial process control to promote consistency, economy, and safety in continuous production processes across many industries. Quality control (QC) is an essential component of quality management (ISO 9000:2005) and involves reviewing production parameters to fulfil quality standards. The Shewart control chart is an important instrument in statistical process control for assuring process stability and improvement (Adewara & Aako, 2020). Quality control has progressed from ensuring that end products fulfilled engineering specifications to controlling process variance for great goods. It entails creating and testing standards to assure the proper completion of products or services. Developing high-quality products and services is critical for global organizations.

Customer expectations establish quality and have an impact on market position (Gbadeyan & Adeoti, 2005). Meeting quality requirements necessitates establishing tolerance limits and concentrating on obtaining high-quality raw materials, equipment, and labour (Orga, 2011). In project management, quality control is assessing completed work to ensure that it is in accordance with the project scope (Phillips & Joseph, 2008). It is a procedure for ensuring stability, assessing performance, and taking corrective actions. The feedback loop is essential in quality control and is relevant in a variety of industries (Juran, 2000).

Recent research on combined distributions in quality control charts has focused on the creation of more robust and flexible strategies for dealing with varied data properties such as non-normality, multivariate dependencies, and autocorrelation. These ideas are useful for practitioners looking to improve process monitoring and control in a variety of sectors.

Chen et al. (2019) introduced a novel control chart framework that models the underlying process using several distributions such as normal, exponential, and Weibull. The graphic becomes more robust and adaptable in spotting process anomalies and shifts by merging these distributions.

Jafari et al. (2020) concentrated on monitoring non-standard process data, which is typical in real-world circumstances. The researchers created mixed distribution control charts, which use a combination of distributions such as normal, lognormal, and gamma to capture the many properties of the data and increase the identification of process fluctuations.

Zhang et al. (2021) introduced multivariate procedures, which incorporate numerous quality parameters at the same time. This study presented joint distribution control charts, which use combined distributions to model the dependencies between numerous variables. These charts give a complete technique for monitoring and diagnosing multivariate process alterations by taking into account the joint behaviour of the variables.

Alwan et al. (2022) presented autocorrelation, which is a prevalent feature in many processes and in which observations depend on past values. This paper presented combination phase II control charts, which combine autoregressive integrated moving average (ARIMA) models with combined distributions. By capturing both time-dependent patterns and distributional properties, the proposed method enables effective monitoring of autocorrelated processes.

The research seeks to monitor the raw materials used in the cement production process using an exponential-pareto distribution model.

2.0 METHODOLOGY

Secondary data created by the company's quality control department for five different constituents (Aluminium, Calcium, Gypsum, Iron, Silicon) utilised in the production of cement in order to check the quality of the cement produced is used for this project. The information ranges from 2007 to 2016.

Exponential-Pareto Distribution

If x has an Exponential-Pareto distribution with the scaling parameter, then x has a normal distribution $\theta_0{}^* = \theta_0{}^{\frac{1}{3.6}}$.

The Exponential-Pareto distribution charts' control limits are given by

$UCL_X = \mu_x + k\lambda_x$

(1)

$$\mu_0 = \lambda\left(1 + {}^{1}/_{\beta}\right) + k\theta_0\sqrt{\lambda\left(1 + {}^{2}/_{\beta}\right) - \lambda\left(1 + {}^{1}/_{\beta}\right)^2}$$

(2)

$$\mu_0 = \lambda\left(1 + {}^{1}/_{\beta}\right) + k\sqrt{\lambda\left(1 + {}^{2}/_{\beta}\right) - \lambda\left(1 + {}^{1}/_{\beta}\right)^2}$$

(3)

$=\boldsymbol{\mu_0 C_u}$

$LCL_X = \mu_x - k\lambda_x$

(4)

$$\mu_0 = \lambda\left(1 + {}^{1}/_{\beta}\right) - k\theta_0\sqrt{\lambda\left(1 + {}^{2}/_{\beta}\right) - \lambda\left(1 + {}^{1}/_{\beta}\right)^2}$$

(5)

$$\mu_0 = \lambda\left(1 + {}^{1}/_{\beta}\right) - k\sqrt{\lambda\left(1 + {}^{2}/_{\beta}\right) - \lambda\left(1 + {}^{1}/_{\beta}\right)^2}$$

(6)

$=\boldsymbol{\mu_0 C_l}$

The inner control limit for EPD mean is also given by

$$\mu_0 = \lambda\left(1 + {}^{1}/_{\beta}\right) + k_2\theta\sqrt{\lambda\left(1 + {}^{2}/_{\beta}\right) - \lambda\left(1 + {}^{1}/_{\beta}\right)^2}$$

(7)

$$\mu_0 = \lambda\left(1 + {}^{1}/_{\beta}\right) - k\sqrt{\lambda\left(1 + {}^{2}/_{\beta}\right) - \lambda\left(1 + {}^{1}/_{\beta}\right)^2}$$

(8)

$=\boldsymbol{\mu_0 C_u}$

In the above,K_1andK_2 $(K_1 > K_2)$ are control coefficients to be determined by considering the target in-control ARL, say r_0

Then the average run length of the distribution is given as

$$\text{ARL}=\frac{1}{1-P_{in}{}^1} \quad (9)$$

and the standard deviation of the average run length is

$$\text{SDRL} = \sqrt{\frac{P_{in}{}^1}{\left[1-P_{in}{}^1\right]^2}} \quad (10)$$

The Average Run Length (ARL) metric is used to evaluate the performance of control charts, either alone or in conjunction with other metrics such as the Cumulative Sum (CUSUM) and Cumulative Poisson (CP) charts. ARL is the average number of in-control observations that occur before a change in process level or an out-of-control observation. In practise, ARL is frequently calculated in conjunction with another parameter known as the Control Limit Index (CLI). Higher ARL and CLI values are expected when a process functions consistently over time and remains statistically in control. When the process deviates or is deemed out of control, it is preferable to have lower ARL and CLI levels.

The statistical software used in the analysis of this research work is R package. The code used in the computation is gotten from (Kimakova, 2021).

3.0 RESULTS

Table 1: Descriptive Analysis of the five raw materials

S/N	*Statistic*	*Aluminium*	*Calcium*	*Gypsum*	*Iron*	*Silicon*
1	N	360	360	360	360	360
2	Mean	32.62	31.56	32.54	33.69	36.03
3	Variance	402.69	327.83	282.04	447.14	515.93
4	StandardDev	20.07	18.10	19.55	21.15	22.71
5	Median	24.02	28.02	26.85	29.59	31.52
6	Minimum	10.01	10.08	10.11	10.11	10.09
7	Maximum	97.26	99.23	98.51	98.51	99.23
8	Range	87.25	89.15	88.40	88.40	89.14
9	Skewness	1.0931	1.0472	1.0718	1.0309	0.9619
10	Kurtosis	0.6907	0.9222	0.7854	0.4944	0.1803

11	Standard Err.	20.0663	18.1060	16.7940	21.1455	22.7140

Sources: Researchers Analysis Output, 2022

The analysis of this study is triple for each constituent (table 1) and the various statistics for each is as presented above.

Table 2: Average Run Length of estimated model parameter and MSE of EPD

Sample Size	**Estimate** θ	α	**MSE** θ	α
50	0.580	0.640	0.597	0.018
100	0.520	0.530	0.031	0.006
150	0.480	0.510	0.026	0.003
200	0.620	0.540	0.009	0.002
250	0.560	0.540	0.003	0.001

Table 2 shows the average run length and mean square error of the Exponential Pareto distribution for various sample sizes. When the average run length of the estimated model parameter and the mean square error are compared, the mean square error reduces as the size of the raw material sample rises. The associated Average Run Length (ARL) for a Shewart chart with the normal 3-sigma limit and a probability of 0.05, representing the likelihood of a single point slipping outside the control limit when the process is in control, is computed as 1/0.05, resulting in an ARL of 20.

Table 3: MLE of Exponential Pareto distribution using SE

Datasets	parameters	Estimate	SE	Log-like
Aluminium	x_m	1	2.34	-151.837
	θ	141.218	168.235	-23.45
	α	3.297	117.67	-12.45
Calcium	x_m	0.11	123.45	-1.616474
	θ	-0.888	0.111	-1.352
	α	0.419	0.059	-2.4523
Gypsum	x_m	1.34	0.345	-167.084
	θ	0.298	0.054	-223.333
	α	1.345	0.223	-126.359
Iron	x_m	11779.33	1043.882	-23.343
	θ	866.907	7675.908	-22.34
	α	1.234	45.65	-134.937
Silicon	x_m	-0.696	0.171	-124.45
	θ	0.606	0.087	-125.34
	α	1	0.345	-138.235
		0.46	0.078	-124.56

Table 4: X-Chart control limit for Exponential Pareto Distribution

Control limit	**Aluminium**	**Calcium**	**Gypsum**	**Iron**	**Silicon**
UCL	47.60622	46.12795	48.1012	108.239	19.2119
LCL	16.43533	16.98611	16.9698	18.0169	52.8454

Table 5: R Control Chart limit for Exponential Pareto Distribution

Control limit	**Aluminium**	**Calcium**	**Gypsum**	**Iron**	**Silica**

UCL	100.6622	15.5217	16.5814	18.0169	17.9141
LCL	16.6021	94.1102	100.5352	109.239	108.615

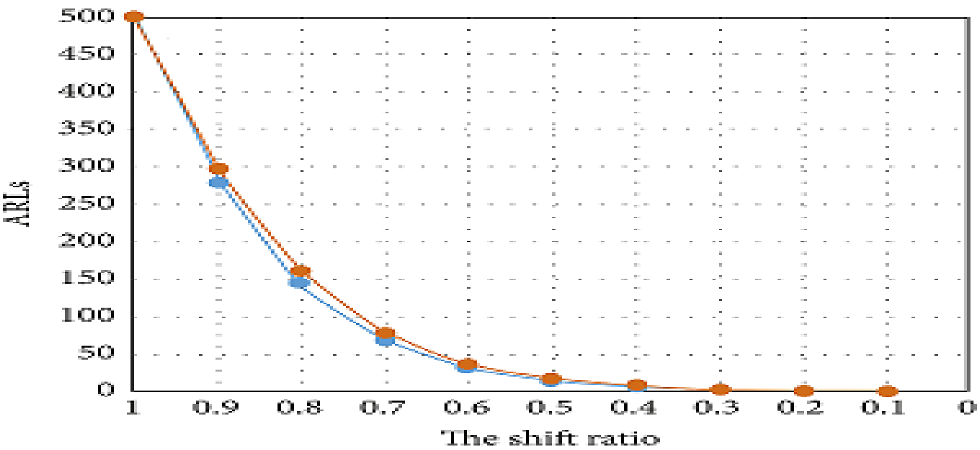

Figure 1: The ARL value for Exponential-Pareto distribution model

Figure 1 shows that the shift in ratio diminishes when the values of the Average Run Length (ARL) distribution grow at a constant interval. This data can be used to evaluate the effectiveness of a single control chart or a set of control charts. When the shift is minor (less than 2.5), a CUSUM chart will notice it before the control chart.

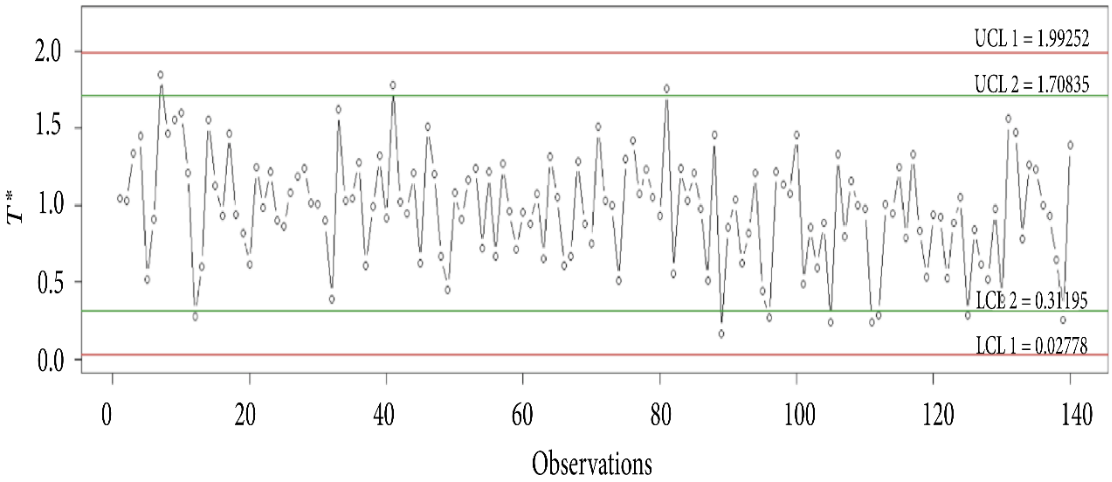

Figure 2: Control Chart of Exponential-Pareto Distribution

The control chart for the Exponential-Pareto distribution in figure 2 shows that none of the points are outside the control limit, indicating that when the distribution is applied to all of the constituents, the performance is in control, with no sign for any assignable causes of variation in the model estimated.

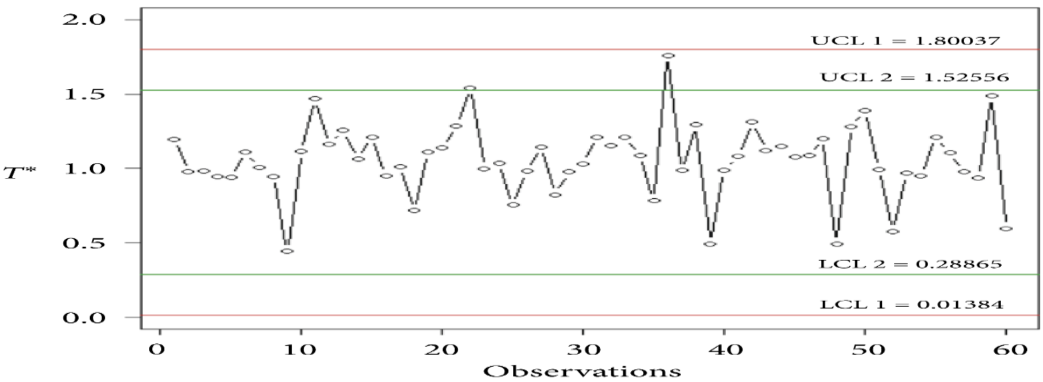

Figure 3: The control Chart for variance of Exponential-Pareto Distribution

Figure 3 shows that the variance of the Exponential-Pareto distribution indicates that none of the points fall outside the control limit, implying that when the distribution is applied to all constituents, the performance is in control, with no sign for any assignable causes of variation in the estimated model.

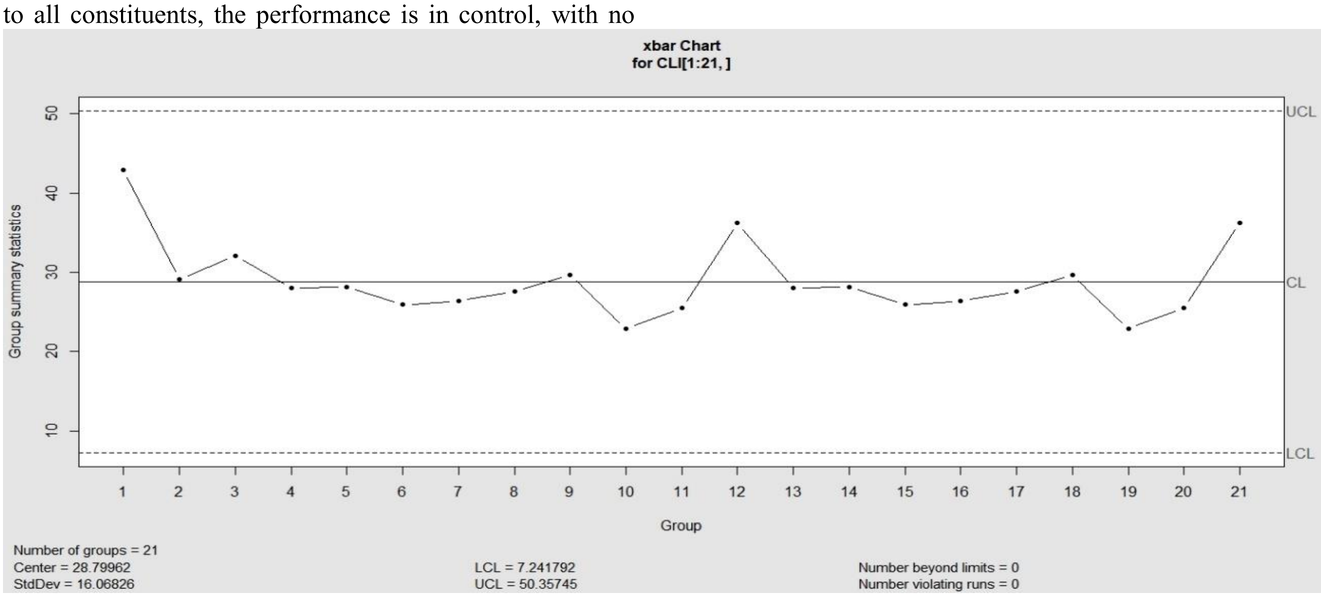

Figure 4: X-bar Chart for Control Limit Interval of Exponential-Pareto Distribution

The X-bar Chart (figure 4) for control limit interval indicates that all of the points are inside the control limit and that there are no assignable reasons of variance in the estimated model.

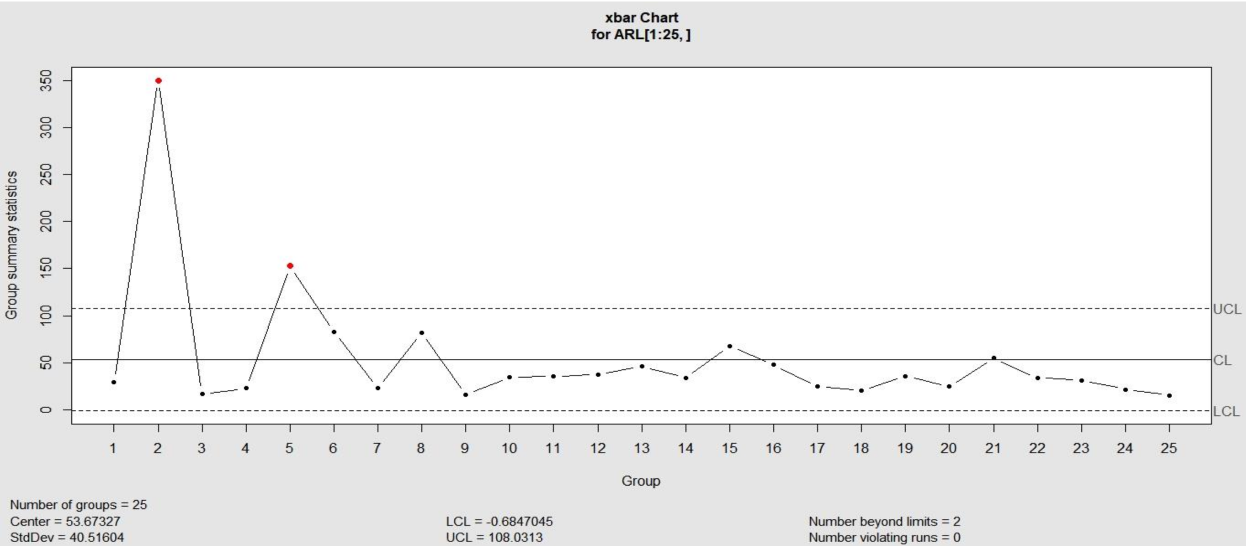

Figure 5: X- bar chart of Average run length Exponential-Pareto Distribution

Figure 5 depicts the mean of the average run length of EPD and reveals that two of the points are outside the control limit, causing the ARL chart to be statistically unstable.

Table 6: ARL, CLI and CP for mean, variance and standard Deviation of Raw Materials

	Statistic	***Aluminium***	***Calcium***	***Gypsum***	***Iron***	***Silicon***
	Mean	32.62	31.56	32.54	33.69	36.03
CLI	**variance**	402.69	372.8	282.04	447.14	515.93
	SD	17.995	18.1	19.55	21.15	22.71
	Mean	31.342	13.435	23.34.7	26.231	34.54
ARL	**variance**	372.83	112.8	223.76	234.87	34.87
	SD	32.82	32.43	65.78	211.12	112.34
	Mean	34.12	12.6	32.343	22.12	34.31
CP	**variance**	231.2	31.556	12.671	112.23	32.0202
	SD	19.345	16.825	17.9737	19.412	19.418

Discussion

We used the average run length (ARL), control limit interval (CLI), and process capability (Cp) of each element to estimate the parameters of Exponential-Pareto distribution models utilising their conditional distributions. The table below shows the findings for the parameters of each constituent utilised in the distribution for estimate.

The generated data, as shown in table 6, revealed that the EPD for the control Limit Interval (CLI) of calcium is the lowest when compared to other constituents, and the variance of silicon is the greatest CLI while the standard deviation of calcium is the lowest.

The average run length (ARL) of the EPD of each constituent demonstrates that aluminium has the greatest mean and iron has the biggest standard deviation.

For EP distribution, we conducted a Monte Carlo simulation analysis. To perform, several samples of each constituent's size were employed. We generated random samples with initial values of = 0.5 and = 0.5, and maximum likelihood estimators are derived using these parameters. The procedure is then repeated 360 times. For the estimates, the mean and mean squared errors (MSEs) are calculated. It is determined that the generated estimations are extremely near to the true values of the parameters. As a result, it demonstrates that the estimating technique is sufficiently accurate. Furthermore, it is investigated if the estimated MSEs decrease consistently with increasing sample size. Finally, we have very clearly witnessed the correctness of the estimating methods.

4.0 CONCLUSION

The purpose of this research is to look into the suitability of the Exponential-Pareto (EP) distribution for drawing control charts. The EP distribution model parameters are determined using the maximum likelihood estimation approach, and its performance is evaluated using a simulation study. Furthermore, the EP distribution is compared to other models using five real-world datasets generated from line one's crusher mill. When compared to

the other models investigated in this work, the EP distribution appears to provide a better fit to the data.
A simulation technique is used to evaluate the shift detection capabilities utilising the R programming language and software. For various mean shifts, control chart coefficients are determined, and different values of Average Run Length (ARL) and Standard Deviation of Run Length (SDRL) are created. The ARL and SDRL figures show that the proposed control chart outperforms existing charts. The process is statistically stable since the ARL, Control Limit Index (CLI), and Cumulative Poisson (CP) production operations are under control, with all data points falling inside the control limits. As a result, the article advises that exponential-pareto be used to measure the quality of raw materials for manufacturing.

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